

钱学森

力学手稿

6

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西安交通大学出版社
XI'AN JIAOTONG UNIVERSITY PRESS

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出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长,学习他的科学思想和卓越风范,展示其丰功伟绩和人格魅力,西安交通大学举办了“纪念钱学森诞辰100周年”系列活动:作为制片方之一,参与西部电影集团摄制传记故事片《钱学森》;与中央电视台合作,出品纪录片《实验班的故事——沿着钱学森走过的路》;扩建钱学森生平业绩展馆,向校内外开放;举办钱学森科学与教育思想研讨会;出版发行《钱学森力学手稿》、《钱学森年谱(初编)》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿,这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风,是钱老勇攀科学高峰和严谨治学的集中体现。这里,我们将部分原稿整理汇集成册,出版《钱学森力学手稿》,作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷,包含两部分内容。第一部分是草稿,包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时,为了求得圆柱壳体的临界压力,在有关的五百多页草稿中,对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算,最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表,每个数字有效位数都长达八位,在手摇机械式计算机作为主要计算工具的年代,这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿,以航空航天工程为核心,涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下,由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助,在此深表感谢。

*Preliminary Calculation of
Circular Cylinder (V)*

$$\frac{u}{R} = (f_0 + \frac{1}{4}f_1) + \frac{1}{2}f_1 \left[\cos \frac{mX}{R} \cos \frac{mY}{R} + \frac{1}{4} \cos \frac{2mX}{R} + \frac{1}{4} \cos \frac{2mY}{R} \right] \\ + \frac{1}{4}f_2 \left[\cos \frac{2mX}{R} + \cos \frac{2mY}{R} \right]$$

$$\frac{u}{R} = (f_0 + \frac{1}{4}f_1) + \frac{1}{2}f_1 \cos \frac{mX}{R} \cos \frac{mY}{R} + \frac{1}{4}(\frac{1}{2}f_1 + f_2) \cos \frac{2mX}{R} + \frac{1}{4}(\frac{1}{2}f_1 + f_2) \cos \frac{2mY}{R}$$

$$\frac{\partial u}{\partial y} = -m \left[\frac{1}{2}f_1 \cos \frac{mX}{R} \sin \frac{mY}{R} + \frac{1}{2}(\frac{1}{2}f_1 + f_2) \sin \frac{2mY}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 u}{\partial x^2} = -\left(\frac{m}{R}\right)^2 \left[\frac{1}{2}f_1 \cos \frac{mX}{R} \cos \frac{mY}{R} + (\frac{1}{2}f_1 + f_2) \cos \frac{2mX}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 u}{\partial y^2} = -\left(\frac{m}{R}\right)^2 \left[\frac{1}{2}f_1 \cos \frac{mX}{R} \cos \frac{mY}{R} + (\frac{1}{2}f_1 + f_2) \cos \frac{2mY}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 u}{\partial x \partial y} = +\left(\frac{m}{R}\right)^2 \left[\frac{1}{2}f_1 \sin \frac{mX}{R} \sin \frac{mY}{R} \right]$$

$$\Delta \phi = E \left(\frac{m}{R}\right)^2 \left[m^2 \left\{ -\frac{1}{8}f_1^2 \left(\cos \frac{2mX}{R} + \cos \frac{2mY}{R} \right) - \frac{1}{4}f_1 \left(\frac{1}{2}f_1 + f_2 \right) \left(\cos \frac{mX}{R} + \cos \frac{3mX}{R} \right) \right. \right. \\ \left. \left. - \frac{1}{4}f_1 \left(\frac{1}{2}f_1 + f_2 \right) \cos \frac{mY}{R} \left(\cos \frac{mY}{R} + \cos \frac{3mY}{R} \right) - \left(\frac{1}{2}f_1 + f_2 \right)^2 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \right\} \right. \\ \left. + \frac{1}{2}f_1 \cos \frac{mX}{R} \cos \frac{mY}{R} + \left(\frac{1}{2}f_1 + f_2 \right) \cos \frac{2mX}{R} \right]$$

$$= E \left(\frac{m}{R}\right)^2 \left[- \left\{ \frac{1}{8}f_1^2 - \left(g - \frac{1}{2}f_1 \right) \right\} \cos \frac{2mX}{R} - \frac{1}{8}f_1^2 \cos \frac{2mY}{R} \right.$$

$$- \left\{ \frac{1}{2}f_1 \left(g - \frac{1}{2}f_1 \right) - \frac{1}{4}f_1 \right\} \cos \frac{mX}{R} \cos \frac{mY}{R} - \frac{1}{4}f_1 \left(g - \frac{1}{2}f_1 \right)^2 \cos \frac{3mX}{R} \cos \frac{mY}{R} \\ \left. - \frac{1}{4}f_1 \left(g - \frac{1}{2}f_1 \right)^2 \cos \frac{mX}{R} \cos \frac{3mY}{R} - \left(g - \frac{1}{2}f_1 \right)^2 m^2 \cos \frac{2mX}{R} \cos \frac{2mY}{R} \right]$$

$$F = E \left(\frac{R}{m} \right)^2 \left[-\frac{1}{16} \left\{ \frac{1}{8} \rho_1^2 m^2 - \left(q - \frac{1}{2} \rho_1 \right) \right\} \cos \frac{2mR}{R} - \frac{1}{128} \rho_1^2 m^2 \cos \frac{2mR}{R} - \frac{1}{8} \left\{ \frac{1}{2} \rho_1 \left(q - \frac{1}{2} \rho_1 \right) m^2 - \frac{1}{2} \rho_1 \int \cos \frac{mR}{R} \cos \frac{mR}{R} \right. \right. \\ \left. \left. - \frac{1}{400} \rho_1 \left(q - \frac{1}{2} \rho_1 \right) m^2 \cos \frac{3mR}{R} \cos \frac{mR}{R} - \frac{1}{400} \rho_1 \left(q - \frac{1}{2} \rho_1 \right) m^2 \cos \frac{mR}{R} \cos \frac{3mR}{R} - \frac{1}{8} \left(q - \frac{1}{2} \rho_1 \right) m^2 \cos \frac{2mR}{R} \cos \frac{2mR}{R} \right\} \right]$$

$$\cdot \sigma_x + \sigma_y = E \left[\frac{1}{4} \left\{ \frac{1}{8} \rho_1^2 m^2 - \left(q - \frac{1}{2} \rho_1 \right) \right\} \cos \frac{2mR}{R} + \frac{1}{32} \rho_1^2 m^2 \cos \frac{2mR}{R} + \frac{1}{2} \left\{ \frac{1}{2} \rho_1 \left(q - \frac{1}{2} \rho_1 \right) m^2 - \frac{1}{2} \rho_1 \int \cos \frac{mR}{R} \cos \frac{mR}{R} \right. \right. \\ \left. \left. + \frac{1}{40} \rho_1 \left(q - \frac{1}{2} \rho_1 \right) m^2 \cos \frac{3mR}{R} \cos \frac{mR}{R} + \frac{1}{40} \rho_1 \left(q - \frac{1}{2} \rho_1 \right) m^2 \cos \frac{mR}{R} \cos \frac{3mR}{R} + \frac{1}{8} \left(q - \frac{1}{2} \rho_1 \right) m^2 \cos \frac{2mR}{R} \cos \frac{2mR}{R} \right\} \right]$$

$$\lambda + \gamma \frac{\sigma}{E} - \frac{1}{2} m^2 \left[\frac{1}{16} \rho_1^2 + \frac{1}{8} \left(q - \frac{1}{2} \rho_1 \right)^2 \right] + \left(\rho_0 + \frac{1}{4} \rho_1 \right) = 0$$

$$\boxed{K = -4 \left(\frac{\sigma}{E} \right)^2 - m^2 \frac{\sigma}{E} \left[\frac{1}{4} \rho_1^2 + \frac{1}{2} \left(q - \frac{1}{2} \rho_1 \right)^2 \right]}$$

$$\sigma_1 = \frac{1}{8} \left\{ \frac{1}{8} \rho_1^2 m^2 - \left(q - \frac{1}{2} \rho_1 \right) \right\}^2 + \frac{1}{512} \rho_1^4 m^4 + \frac{1}{16} \left\{ \rho_1 \left(q - \frac{1}{2} \rho_1 \right) m^2 - \rho_1 \right\}^2 + \frac{1}{800} \rho_1^2 m^4 \left(q - \frac{1}{2} \rho_1 \right)^2 \\ + \frac{1}{64} m^4 \left(q - \frac{1}{2} \rho_1 \right)^4$$

$$p_1 = \frac{1}{8} \left\{ \frac{1}{64} p_1^4 m^4 - \frac{1}{4} p_1^2 m^2 \left(q - \frac{1}{2} p_1 \right) + \left(q - \frac{1}{2} p_1 \right)^2 + \frac{1}{64} p_1^4 m^4 + \frac{1}{2} p_1^2 m^2 \left(q - \frac{1}{2} p_1 \right)^2 - p_1^2 m^2 \left(q - \frac{1}{2} p_1 \right) \right. \\ \left. + \frac{1}{2} p_1^2 + \frac{1}{100} p_1^2 m^4 \left(q^2 - p_1^2 + \frac{1}{4} p_1 \right) + \frac{1}{8} m^4 \left(q^4 - 2 q p_1^2 + \frac{3}{2} p_1^2 q^2 - \frac{1}{2} p_1^3 + \frac{1}{16} p_1^4 \right) \right\}$$

$$= \frac{1}{8} \left\{ \frac{1}{64} p_1^4 m^4 - \frac{1}{4} p_1^2 m^2 + \frac{1}{8} p_1^2 m^2 + q^2 - p_1^2 + \frac{1}{4} p_1^2 + \frac{1}{64} p_1^4 m^4 + \frac{1}{2} p_1^2 m^2 \left(q - \frac{1}{2} p_1 \right)^2 - \frac{1}{2} p_1^2 m^2 \left(q - \frac{1}{2} p_1 \right) \right. \\ \left. + \frac{1}{8} p_1^2 m^4 - p_1^2 m^2 + \frac{1}{2} p_1^2 m^2 + \frac{1}{2} p_1^2 + \frac{1}{100} p_1^2 m^4 - \frac{1}{100} p_1^3 m^4 + \frac{1}{400} p_1^4 m^4 \right. \\ \left. + \frac{1}{8} p_1^4 m^4 - \frac{1}{4} p_1^2 m^2 + \frac{1}{16} p_1^2 m^4 - \frac{1}{16} p_1^3 m^4 + \frac{1}{128} p_1^4 m^4 \right\}$$

$$= \frac{1}{8} \left[m^4 \left(\frac{1}{64} + \frac{1}{64} + \frac{1}{8} + \frac{1}{128} \right) p_1^4 + \left(-\frac{1}{2} - \frac{1}{100} - \frac{1}{16} \right) p_1^2 q + \left(\frac{1}{2} + \frac{1}{100} + \frac{3}{16} \right) p_1^2 - \frac{1}{4} p_1^2 q^2 + \frac{3}{8} p_1^2 \right] \\ - m^2 \left\{ \left(-\frac{1}{8} - \frac{1}{2} \right) p_1^3 + \left(\frac{1}{4} + 1 \right) p_1^2 q \right\} + \left[\frac{3}{4} p_1^2 - p_1^2 + q^2 \right]$$

$$p_1 = \frac{1}{8} \left[m^4 \left\{ \frac{533}{3200} p_1^4 - \frac{229}{400} p_1^2 q + \frac{229}{400} p_1^2 q^2 - \frac{1}{4} p_1^3 q^3 + \frac{1}{8} p_1^4 \right\} \right. \\ \left. + m^2 \left\{ \frac{3}{8} p_1^3 - \frac{5}{4} p_1^2 q \right\} + \left[\frac{3}{4} p_1^2 - p_1^2 + q^2 \right] \right]$$

$$g_2 = \frac{1}{12(1-v^2)} \left(\frac{t^2}{R} \right) m^4 \left[f_1^2 + 4 \left(g - \frac{1}{2} f_1 \right)^2 \right] = \frac{1}{24(1-v^2)} \left(\frac{t^2}{R} \right) m^4 \left[2f_1^2 + 4g^2 - 4gf_1 \right]$$

$$\eta = \left(\frac{t}{R} \right) = \frac{g}{(R)}$$

$$s = \frac{f_1}{2}$$

$$g_2 = \frac{1}{6(1-v^2)} \left(\frac{t^2}{R} \right) m^4 \left[f_1^2 - 2gf_1 + 2g^2 \right]$$

$$K = -4 \left(\frac{g}{E} \right)^2 - m^2 w \left[\frac{3}{2} f_1^2 - \frac{1}{2} gf_1 + \frac{1}{2} g^2 \right]$$

$$\frac{5R}{E} \gamma \left(\frac{3}{4} g - \frac{1}{2} \right) = \frac{1}{2} \left[(v\eta)^2 \left(\frac{533}{800} g^3 - \frac{667}{400} g^2 + \frac{219}{200} g - \frac{1}{4} \right) + (v\eta) \left(\frac{15}{2} g^2 - \frac{5}{2} g \right) \right. \\ \left. + \left(\frac{3}{2} g - 1 \right) \right] + \frac{1}{3(1-v^2)} \gamma^2 (g-1)$$

$$\frac{5R}{E} \gamma \left(\frac{3}{4} g - 1 \right) = \frac{1}{2} \left[(v\eta)^2 \left(\frac{219}{400} g^3 - \frac{219}{200} g^2 + \frac{3}{4} g - \frac{1}{2} \right) + (v\eta) \left(\frac{5}{2} g^2 + (g-2) \right) \right. \\ \left. + \frac{1}{3(1-v^2)} \gamma^2 (g-2) \right]$$

$$\frac{1}{16} \quad \frac{1}{10} \quad \frac{1}{2}$$

$$\frac{dR}{dt} \gamma (3s-2) = (\gamma \eta)^2 \left(\frac{533}{1600} s^3 - \frac{617}{800} s^2 + \frac{279}{400} s - \frac{1}{8} \right) + (\gamma \eta) \left(\frac{15}{16} s^2 - \frac{5}{4} s \right) + \left(\frac{3}{4} s - \frac{1}{2} \right) + \frac{s^2}{3(1-v^2)} \gamma' (2s-2)$$

$$\frac{dR}{dt} \delta (s-2) = (\gamma \eta)^2 \left(\frac{229}{1600} s^3 - \frac{279}{800} s^2 + \frac{75}{400} s - \frac{1}{8} \right) + (\gamma \eta) \left(\frac{15}{16} s^2 + 0 \right) + \left(\frac{1}{4} s - \frac{1}{2} \right) + \frac{s^2}{3(1-v^2)} \gamma' (s-2)$$

$$0 = (\gamma \eta)^2 \left(\frac{154}{1600} s^4 - \frac{150}{800} s^3 - \frac{56}{400} s^2 - \frac{25}{100} s \right) + (\gamma \eta) \left(\frac{5}{4} s^2 + 0 \right) + (-s + 0) + \frac{s^2}{3(1-v^2)} \gamma' (s^2 - 4s) \\ + (\gamma \eta)^2 \left(\frac{304}{800} s^3 - \frac{408}{400} s^2 + \frac{102}{100} s \right) + (\gamma \eta) \left(\frac{5}{4} s^2 - \frac{5}{2} s \right) + (s) + \frac{s^2}{3(1-v^2)} \gamma' (+4s)$$

$$0 = (\gamma \eta)^2 \left(\frac{77}{800} s^3 + \frac{77}{400} s^2 - \frac{231}{200} s + \frac{37}{100} \right) + (\gamma \eta) \left(\frac{5}{2} s - \frac{5}{2} \right) + \frac{s^2}{3(1-v^2)} \gamma' (s-2)$$

$$\left\{ \frac{77}{800} (\gamma \eta)^2 \right\} s^3 + \left\{ \frac{77}{400} (\gamma \eta)^2 \right\} s^2 + \left\{ -\frac{231}{200} (\gamma \eta)^2 + \frac{5}{2} (\gamma \eta) + \frac{s^2}{3(1-v^2)} \gamma' \right\} s \\ + \left\{ \frac{37}{100} (\gamma \eta)^2 - \frac{5}{2} (\gamma \eta) - \frac{4}{3(1-v^2)} \gamma' \right\} = 0$$

$$\boxed{\gamma = 0.10, \quad \eta = 10, \quad \xi = 10.5051, \quad \gamma\eta = 1}$$

643

$$\lambda = -0.0480814$$

$$0.09625 s^3 + 0.19250 s^2 + 1.352326 s - 1.744652 = 0$$

$$F(s) = s^3 + 2.00000 s^2 + 14.05014 s - 18.12625 = 0$$

$$F'(s) = 3s^2 + 4.00000 s + 14.05014$$

$$F(1.05) = -0.010978$$

$$F'(1.05) = 21.558$$

$$\frac{0.00051}{21.558}$$

$$F(1.05051) = 0$$

$$s^2 + 3.05051 s + 17.2547 = 0 \quad \text{has } \text{here} \text{ Real Root !!!}$$

$$\therefore \boxed{s = 1.05051, \quad s^2 = 1.10357, \quad s^3 = 1.15931}$$

$$\frac{\sigma_R}{Et} = \frac{2}{3(1-\nu^2)} \gamma + \frac{1}{\eta(s-2)} \left\{ \frac{1}{(1\eta)^2} (0.143125 s^3 - 0.34875 s^2 + 0.1875 s - 0.1250) \right. \\ \left. + (\eta\eta) 0.3125 s^2 \right\} + \frac{1}{4\eta}$$

For this particular case

$$\frac{\sigma_R}{Et} = 0.07326 + 10 \left\{ \frac{0.143125 s^3 - 0.03625 s^2 + 0.1875 s - 0.1250}{-0.94949} + 0.25 \right\}$$

$$= 0.07326 + 0.41580 = \underline{\underline{0.4891}}$$

$$\{f\} = 1.05051$$

$$\{f\}^2 = 1.103571$$

$$\begin{aligned} \bar{E} &= 0.23919 + (1.05051)^2 \left[0.004310824(\lambda)^6 - 0.008621148(-\lambda)^5 \right. \\ &\quad \left. + 0.056320149(-\lambda)^4 - 0.010833339(-\lambda) + 0.009121722 \right] \end{aligned}$$

$$= \frac{1.2024}{(0.9502359)}$$

$$\Theta = \frac{0.2653712}{}$$

Check III

644

$$\frac{\eta R}{Et} = \frac{2}{3(1-\nu^2)} \gamma \frac{2s-2}{3s-2} + \frac{1}{\gamma(3s-2)} \left\{ (\gamma\eta)^2 (0.333125 s^3 - 0.65875 s^2 + 0.6925 s - 0.125) \right. \\ \left. + (\gamma\eta) (0.9375 s^2 - 1.250 s) + (0.25 s - 0.5) \right\}$$

$$= \frac{0.2}{3(1-\nu^2)} \frac{0.10102}{1.15153} + \frac{10}{1.15153} \left\{ 0.333125 s^3 + 0.07875 s^2 + 0.1915 s - 0.625 \right\}$$

$$= \underline{0.48907} \quad O.K. \quad \left(\frac{\epsilon R}{t} \right) = 0.9778 \quad \underline{\epsilon} = +0.124455$$

$$\eta = 10 \quad \boxed{\gamma = 0.144 \quad \gamma\eta = 1.44} \quad s = 10.7961, \quad \lambda = 0.0737316$$

$$0.199584 s^3 + 0.399168 s^2 + 1.220163 s - 2.033710 = 0$$

$$F(s) = s^3 + 2.00000 s^2 + 6.113631 s - 10.18974 = 0$$

$$F'(s) = 3s^2 + 4.00000 s + 6.113631$$

$$F(1) = -1.07611$$

$$F'(1) = 13.113631$$

$$F(1.082) = +0.03338$$

$$F'(1.082) = 13.954$$

$$\frac{.00239}{1.07961}$$

$$F(1.07961) = +0.00006, \quad O.K.$$

$$s^2 + 3.07961 s + 9.43841 = 0$$

No more real Root

$$f^2 = 0.020736$$

$$(f_3) = 1.5546384$$

$$(f_3)^2 = 2.4169006$$

$$\bar{E} = 0.0461446 + (10.7951)^2 \left\{ 0.00944101(-\lambda)^4 - 0.014442036(-\lambda)^3 \right.$$

$$\left. + 0.0858297(-\lambda)^2 - 0.0156607(-\lambda) + 0.006603078 \right\}$$

$$= \underline{0.7347181} \quad (0.847512)$$

$$(-) = -0.1503705$$

$$s = 1.07961, \quad s^2 = 1.16555, \quad s^3 = 1.25834$$

645

$$\frac{SR}{Et} = 0.105495 + 6.944444 \left\{ \frac{0.29628s^3 - 0.273168s^2 + 0.388600s - 0.2592}{-0.92039} + 0.25 \right\}$$

$$= 0.105495 + 0.109257 = \underline{\underline{0.2148}}$$

Check

$$\frac{SR}{Et} = \frac{0.288}{2.73} \frac{0.15922}{1.23883} + \frac{6.94444}{1.3883} \left\{ 0.690768s^3 - 0.430704s^2 + 0.396336s - 0.7592 \right\}$$

$$= 0.0135587 + 0.201254 = \underline{\underline{0.214813}}$$

$$\left(\frac{ER}{E} \right) = 0.92992$$

$$\Phi = + 0.167600$$

$$\boxed{\lambda = 2.147 \quad n = 15, \quad \frac{1}{\lambda} = 316} \quad s = 17.1025,$$

$$0.449064s^3 + 0.898128s^2 + 0.0264230s - 1.837870 = 0 \quad \lambda = \underline{\underline{0.1177768}}$$

$$F(s) = s^3 + 2.00000s^2 + 0.0588402s - 4.09268 = 0$$

$$F'(s) = 3s^2 + 4.000s + 0.0588402$$

$$F(1.134) = + 0.004243$$

$$F'(1.134) = 6.4527$$

$$\frac{50}{1.13350}$$

$$F(1.13350) = 0,$$

$$s^2 + 3.13350s + 3.61066 = 0$$

No more
real roots.

$$\gamma^2 = 0.020736, \quad (\beta) = 2.4483600$$

$$(\beta)^2 = 5.9944667$$

$$\begin{aligned} \mathcal{E} = & 0.3350094 + (17.0025)^2 \left\{ 0.023415886 (-1)^4 - 0.046831771 (-1)^3 \right. \\ & \left. + 0.113809542 (-1)^2 - 0.044754594 (-1) + 0.007769092 \right\} \end{aligned}$$

$$= \underline{1.6932169} \quad (5.1061960)$$

$$\Theta = -0.668432$$

$$s = 1.13350 ; s^2 = 1.28482, s^3 = 1.45634 \quad (64)$$

$$f = 0.144 \quad (np) = 2.16$$

$$\frac{OR}{Et} = \frac{0.248}{2.73} \frac{0.26700}{1.40050} + \frac{6.94444}{1.40050} \left\{ 1.55422s^3 - 1.92584s^2 + 1.32425s - 1.083200 \right\}$$

$$= 0.0201122 + 0.558729 = \underline{\underline{0.57884}} \quad \left(\frac{ER}{t} \right) = \underline{\underline{2.2598}} \quad \Phi = -0.461363$$

$$\boxed{f = 0.121 \quad g = 15. \quad (np) = 1.815} \quad s = 16.56975$$

$$0.3170692s^3 + 0.6341383s^2 + 0.7433961s - 2.02259875 = 0 \quad \lambda = \underline{\underline{0.0947359}}$$

$$F(s) = s^3 + 2.00000s^2 + 2.344586s - 6.378414 = 0$$

$$F'(s) = 3s^2 + 4.0000s + 2.344586$$

$$F(1.09) = -0.1516 \quad F'(s) = 10.219$$

14%

$$F(1.10470) = +0.000508 \quad F'(s) = 10.424$$

000049

$$F(1.10465) = 0 \quad s = 1.10465, s^2 = 1.22025, s^3 = 1.34795$$

$$\frac{OR}{Et} = \frac{0.242}{2.73} \frac{0.20930}{1.31395} + \frac{8.264763}{1.31395} \left\{ 1.091327s^3 - 1.127353s^2 + 0.724972s - 0.711728 \right\}$$

$$= 0.014120 + 0.32117 = \underline{\underline{0.3430}}$$

$$\left(\frac{ER}{t} \right) = (1.7252) \quad (2.965971r) \quad \underline{\underline{\tau}} = +0.07851$$

$$\gamma^2 = 0.014641, \quad (\gamma^2) = 2.0049398$$

$$(\gamma^2)^2 = 0.0197136$$

$$\begin{aligned} E = & 0.117649 + (16.56975)^2 \left\{ 0.01570280(\lambda)^4 - 0.031404559(-\lambda)^3 \right. \\ & \left. + 0.120209422(-\lambda)^2 - 0.026187231(-\lambda) + 0.005472126 \right\} \end{aligned}$$

$$= \underline{\underline{1.3379312}}$$

$$\Theta = - \underline{\underline{0.5489065}}$$

$$\boxed{f = 0.100, \quad \eta = 15, \quad (\eta f) = 1.5} \quad \xi = 16.21518 \quad \underline{\underline{647}}$$

$$0.2165625 s^3 + 0.433125 s^2 + 1.158576 s - 2032152 = 0 \quad \lambda = -0.0749409$$

$$F(s) = s^3 + 2.00000 s^2 + 5.349146 s - 9.383674 = 0$$

$$F'(s) = 3s^2 + 4.00000 s + 5.349146$$

$$F(1.08) = -0.013328$$

$$s01012$$

$$F'(s) = 13.169$$

$$F(1.081012) = 0$$

$$s^2 + 3.081012 s + 6.680457 = 0 \quad \text{No more real roots!!!}$$

$$s = 1.081012, \quad s^2 = 1.168587, \quad s^3 = 1.263257$$

$$\frac{\varepsilon R}{Et} = \frac{0.2}{2.73} \frac{0.162024}{1.243036} + \frac{10}{1.243036} \left\{ 0.74953125 s^3 - 0.5259375 s^2 + 0.444375 s - 0.78125 \right\}$$

$$= 0.00954908 + 0.25238 = \underline{\underline{0.2619}}$$

$$\left(\frac{\varepsilon R}{Et} \right) = \left(1.3805 \right) \quad (1.9057403) \quad \xi^2 = 0.01, \quad (\xi) = 1.621518$$

$$(\xi)^2 = 2.6293206$$

$$\xi = 0.065916 + (16.21518)^2 \left\{ 0.01027024 (-\lambda)^4 - 0.02054156 (-\lambda)^3 \right. \\ \left. + 0.08947123 (-\lambda)^2 - 0.015415393 (-\lambda) + 0.005910924 \right\}$$

$$= \left(\frac{1.4400844}{1} \right)$$

$$\Theta = -0.2443597 \quad \Phi = +0.35147$$

$$\begin{array}{l} \gamma = 0.121, \quad \gamma_1 = 10, \quad \gamma_2 = 121 \\ (1\gamma)^2 = 1.4641 \end{array} \quad \begin{array}{l} \xi = 1063807 \text{ €} \\ \lambda = -0.0599799 \end{array}$$

$$0.140919625 \xi^3 + 0.28183925 \xi^2 + 1.3446905 \xi - 19170950 = 0$$

$$F(\xi) = \xi^3 + 2.000000 \xi^2 + 9.542253 \xi - 13618365 = 0$$

$$F'(\xi) = 3\xi^2 + 4.000000 \xi + 9.542253$$

$$F(1.062) = -0.03103$$

$$F'(1.062) = 13.173$$

$$-0.001807$$

$$F(1.063807) = \text{O.K.}$$

$$\xi = 1.063807, \quad \xi^2 = 1.131685, \quad \xi^3 = 1.203294$$

$$\frac{\xi R}{E L} = \frac{0.242}{2.73} \frac{0.127614}{1.191421} + \frac{1.264463}{1.91421} \left\{ 0.4877283 \xi^3 - 0.1229209 \xi^2 + 0.25870925 \xi - 0.683525 \right\}$$

$$= 0.00949480 + 0.279340 = \underline{0.2888}$$

$$\left(\frac{\xi R}{E L} \right) = \overset{(0.7781298)}{0.8824...}$$

$$\begin{array}{l} \xi^2 = 0.014641, \quad \xi^3 = 1.472065 \\ (1\xi)^2 = 1.151906 \end{array}$$

$$\begin{aligned} \bar{E} &= 0.0834054 + (10.63807)^2 \left\{ 0.006472268 (-1)^4 - 0.012944536 (-1)^3 \right. \\ &\quad \left. + 0.068706006 (-1)^2 - 0.011952234 (-1) + 0.002591422 \right\} \\ &= \underline{0.1890517} \end{aligned}$$

$$\Theta = + \underline{0.1418157}$$

$$\Phi = + 0.179149$$

$$\underline{\eta = 0.100, \quad \eta = 12.50}$$

$$(\eta V) = 1.250$$

$$(\eta V)^2 = 1.5625$$

$$\xi = 12.311325 \underline{\underline{049}}$$

$$0.150390625 s^3 + 0.30078125 s^2 + 1.3276365 s - 1.936527 = 0 \quad \lambda = -0.0609539$$

$$F(s) = s^3 + 200000 s^2 + 8.827934 s - 12.876647 = 0$$

$$F'(s) = 3s^2 + 400000 s + 8.827934$$

$$F(1.062) = -0.047923$$

$$0.0291$$

$$F'(1.062) = 16459$$

$$F(1.06491) = +0.000018, \quad 0.5$$

$$s = 1.064910, \quad s^2 = 1.134033, \quad s^3 = 1.207643$$

$$\frac{\sigma R}{Et} = \frac{0.2}{2.73} \frac{0.129820}{1.194730} + \frac{10}{1.194730} \left\{ 0.5205078 s^3 - 0.1699219 s^2 + 0.2773438 s - 0.6953125 \right\}$$

$$= 0.007960 + 0.30769 = \underline{\underline{0.30815}}$$

$$\left(\frac{\varepsilon R}{t} \right) = \underline{\underline{1.0759}} \quad (1.1575608)$$

$$\gamma^2 = 0.01, \quad (1/3) = 1.3311325$$

$$(1/3)^2 = 1.7719270$$

$$\begin{aligned} \bar{E} = & 0.0952146 + 1/3.311325 \left\{ 0.001921590 (-1)^4 - 0.013243180 (-1)^3 \right. \\ & \left. + 0.170744222 (-1)^2 - 0.011867074 (-1) + 0.002119617 \right\} \end{aligned}$$

$$= \underline{\underline{1.2747}}$$

$$\Delta \Theta = + \underline{\underline{0.1011949}}$$

$$\Phi = + 0.307273$$

$$\boxed{\gamma = 0.121, \quad \eta = 12.5}$$

$$\gamma^2 = 0.014641$$

$$(\eta\gamma) = 1.5125$$

$$(\eta\gamma)^2 = 2.28765625 \quad \xi = 13.53684 \quad \underline{\underline{6.50}}$$

$$0.2201869 \xi^3 + 0.4403738 \xi^2 + 1.1497330 \xi - 2.0412067 = 0 \quad \lambda = -0.0765939$$

$$F(\xi) = \xi^3 + 2.00000 \xi^2 + 5.221623 \xi - 9.270337 = 0$$

$$F'(\xi) = 3\xi^2 + 4.00000 \xi + 5.221623$$

$$F(1.0129) = -0.000610$$

$$\underline{\underline{0.00047}}$$

$$F'(1.0129) = 12.07124$$

$$F(1.012947) = 0$$

$$\xi = 1.012947, \quad \xi^2 = 1.172774, \quad \xi^3 = 1.270052$$

$$\frac{QR}{Et} = \frac{0.242}{2.73} \frac{0.165894}{1.248841} + \frac{8.2644628}{1.248841} \left\{ 0.7620754 \xi^3 - 0.5465560 \xi^2 + 0.4550152 \xi - 0.789570 \right\}$$

$$= 0.0117754 + 0.222945 = \underline{\underline{0.23472}}$$

$$\frac{\xi R}{t} = \underline{\underline{1.1761}}^{(1.3832112)}$$

$$(\xi\gamma) = 1.6329521$$

$$(\xi\gamma)^2 = 2.6429051$$

$$\xi = 0.0550935 + (13.53684)^{-1} \left\{ 0.010420098 (-\lambda)^4 - 0.020910196 (-\lambda)^3 + 0.049863022 (-\lambda)^2 - 0.01661881 (-\lambda) + 0.00601249 \right\}$$

$$= \underline{\underline{1.031318}}$$

$$\Theta = - \underline{\underline{0.2542031}}$$

$$\Phi = + 0.239605$$

$$\boxed{\gamma = 0.144 \quad \eta = 12.5}$$

$$\gamma^2 = 0.020736$$

$$\eta^2 = 1.8025$$

$$(\eta\gamma)^2 = 3.2400$$

$$\xi = 13.60915 \quad \underline{\underline{651}}$$

$$\lambda = 0.0948031$$

$$0.3118500 \rho^3 + 0.6237000 \rho^2 + 0.7729912 \rho - 2.0355824 = 0$$

$$F(\rho) = \rho^3 + 2.000000 \rho^2 + 2.428728 \rho - 6.522711 = 0$$

$$F'(\rho) = 3\rho^2 + 4.000000 \rho + 2.428728$$

$$F(1.1047) = -0.000332$$

$$0.000316$$

$$F'(1.1047) = 10.5586$$

$$\rho = 1.104732, \quad \rho^2 = 1.220432, \quad \rho^3 = 1.346250$$

$$\frac{\sigma R}{Et} = \frac{0.288}{2.73} \frac{0.209464}{1.314196} + \frac{6.944444}{1.314196} \left\{ 1.079325 \rho^3 - 1.09485 \rho^2 + 0.759900 \rho - 0.9050000 \right\}$$

$$= 0.016814 + 0.282111 = \underline{\underline{0.298925}}$$

$$\left(\frac{\epsilon R}{t} \right) = \frac{1.4394}{(2.0712724)}$$

$$\eta^2 = 1.9115176$$

$$(\eta\gamma)^2 = 3.9542022$$

$$\bar{G} = 0.0896943 + (13.60915)^2 \left\{ 0.015446102 (-1)^4 - 0.030472205 (-1)^3 \right. \\ \left. + 0.119337152 (-1)^2 - 0.026215561 (-1) + 0.006130651 \right\}$$

$$= \underline{\underline{0.98458}}$$

$$\angle \Theta = \underline{\underline{-0.524723}}$$

$$\Phi = +0.061204$$

$$\boxed{\lambda = 0.100 \quad \eta = 17.5}$$

$$(\eta/\lambda) = 1750$$

$$(\eta/\lambda)^2 = 3.0625$$

$$\xi = 19.2293325 \quad \underline{\underline{652}}$$

$$\lambda = -0.0899320$$

$$0.294765625 s^3 + 0.58953125 s^2 + 0.8451385 s - 2.0315270 = 0$$

$$F(s) = s^3 + 2.000000 s^2 + 2.867154 s - 6.891068 = 0$$

$$F'(s) = 3s^2 + 4.000000 s + 2.867154$$

$$F(1.0968) = -0.000248$$

$$000019$$

$$F'(1.0968) = 10.88$$

$$s = 1.096819, \quad s^2 = 1.202903, \quad s^3 = 1.321217$$

$$\frac{OR}{Et} = 0.007326007 \times \frac{0.197638}{1.296457} + \frac{10}{1.296457} \left\{ 1.0201953 s^3 - 0.9892969 s^2 + 0.6985938 s - 0.8828125 \right\}$$

$$= 0.0111681 + 0.338198 = \underline{\underline{0.34938}}$$

$$\left(\frac{\varepsilon p}{t} \right) = \underline{\underline{1.8935}} \quad (13.5753433)$$

$$(\eta/\lambda) = 19.2293325$$

$$(\eta/\lambda)^2 = 3.6976725$$

$$\bar{G} = 0.1220664 + (19.2293325)^2 \left\{ 0.0144 - 2.33 (-1)^4 - 0.04888066 (-1)^3 + 0.112763456 (-1)^2 - 0.023204843 (-1) + 0.005584763 \right\}$$

$$= \underline{\underline{1.74529}}$$

$$\Delta \Theta = \underline{\underline{-0.5132152}}$$

$$\Phi = + 0.211094$$

$$K = -4\left(\frac{5}{E}\right)^2 - m^2 \frac{m}{E} \left[\frac{3}{8} f_1^2 + \frac{1}{2} f_1 f_2 + \frac{4}{9} f_2^2 \right]$$

$$f_0 = \frac{4}{9} \left[m^4 \left(\frac{533}{3200} f_1^4 + \frac{229}{400} f_1^3 f_2 + \frac{229}{400} f_1^2 f_2^2 + \frac{1}{4} f_1 f_2^3 + \frac{1}{8} f_2^4 \right) - m^2 \left(\frac{5}{8} f_1^3 + \frac{1}{4} f_1^2 f_2 + \frac{1}{8} f_1 f_2^2 + \frac{1}{27} f_2^3 \right) \right]$$

$$f_2 = \frac{1}{6(1-\nu^2)} \left(\frac{1}{R} \right)^2 m^4 \left[f_1^2 + 2 f_1 f_2 + 9 f_2^2 \right]$$

$$\frac{6R}{E} \gamma \left(\frac{1}{2} \lambda + \frac{3}{4} \right) = \frac{1}{8} \left[(15R)^2 \left(\frac{1}{4} \lambda^3 + \frac{229}{200} \lambda^2 + \frac{667}{400} \lambda + \frac{533}{800} \right) - (15R) \left(\frac{5}{2} \lambda + \frac{15}{8} \right) + \frac{1}{3(1-\nu^2)} \lambda^2 (5+1) \right] + \left(\lambda + \frac{3}{2} \right)$$

$$\frac{6R}{E} \gamma \left(\lambda + \frac{1}{4} \right) = \frac{1}{8} \left[(15R)^2 \left(\frac{1}{2} \lambda^3 + \frac{3}{4} \lambda^2 + \frac{229}{200} \lambda + \frac{227}{400} \right) - (15R) \left(\frac{1}{2} + \frac{5}{4} \right) + \frac{1}{3(1-\nu^2)} \lambda^2 (2\lambda+1) \right]$$

$$\frac{6R}{E} \gamma (2\lambda+3) = (15R)^2 \left(\frac{1}{8} \lambda^3 + \frac{229}{400} \lambda^2 + \frac{667}{800} \lambda + \frac{533}{1600} \right) - (15R) \left(\frac{5}{4} \lambda + \frac{15}{16} \right) + \left(\frac{1}{2} + \frac{3}{4} \right) + \frac{1}{3(1-\nu^2)} \lambda^2 (2\lambda+2)$$

$$\frac{6R}{E} \gamma (2\lambda+1) = (15R)^2 \left(\frac{1}{8} \lambda^3 + \frac{75}{400} \lambda^2 + \frac{229}{800} \lambda + \frac{229}{1600} \right) - (15R) \left(0 + \frac{5}{16} \right) + \left(\frac{1}{2} + \frac{1}{4} \right) + \frac{1}{3(1-\nu^2)} \lambda^2 (2\lambda+1)$$

$$\lambda = \frac{1}{2} \quad \quad \quad \lambda = \frac{1}{2} \quad \quad \quad \lambda = \frac{1}{2}$$

$$\gamma = m^2 \frac{1}{R}$$

$$0 = (\nu \xi)^2 \left(\frac{77}{100} \lambda^3 + \frac{231}{100} \lambda^2 + \frac{77}{400} \lambda - \frac{77}{400} \right) - (\nu \xi) \left(\frac{5}{2} \lambda^2 + \frac{5}{2} \lambda \right) - \frac{2}{3(1-\nu)} \nu^2 (2\lambda + 1)$$

$$\left\{ \frac{77}{100} (\nu \xi)^2 \right\} \lambda^3 + \left\{ \frac{231}{200} (\nu \xi)^2 - \frac{5}{2} (\nu \xi) \right\} \lambda^2 + \left\{ \frac{77}{400} (\nu \xi)^2 - \frac{5}{2} (\nu \xi) - \frac{4}{3(1-\nu)} \nu \right\} \lambda - \left\{ \frac{77}{800} (\nu \xi)^2 + \frac{2}{3(1-\nu)} \nu^2 \right\} = 0$$

$$\frac{\sigma_R}{E_0} = \frac{2\nu}{3(1-\nu)} \frac{2(\lambda+1)}{2\lambda+3} + \frac{1}{\nu(2\lambda+3)} \left\{ 0.115000(\nu \xi)^2 \lambda^3 + 0.672500(\nu \xi)^2 \lambda^2 + [0.658750(\nu \xi)^2 - 12500\nu \xi + 0.5] \lambda + [0.3331250(\nu \xi)^2 - 0.9375(\nu \xi) + 0.2500] \right\}$$

$$\boxed{\gamma = 0.121, \quad \xi = 8.1}$$

$$(\gamma\xi) = 0.968$$

655

$$(\gamma\xi)^2 = 0.937024$$

$$0.7215015 \lambda^3 - 1.3377373 \lambda^2 - 1.2610749 \lambda - 0.10091456 = 0$$

$$F(-\lambda) = \lambda^3 + 1.854084 \lambda^2 - 3.133816 \lambda + 0.1398161 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 3.908168 \lambda - 3.133816$$

$$F(0.04) = +0.017544$$

0.0590

$$F'(0.04) = 2.9727$$

$$F(0.04590) = +0.0000168$$

$$F'(0.04590) = 2.978$$

$$F(0.0459091) = 0.00$$

$$\lambda = -0.0459091, \quad \lambda^2 = +0.0021076, \quad \lambda^3 = -0.0000968$$

$$\frac{\sigma R}{Et} = \frac{0.242}{2.23} \frac{1.9081818}{2.9081118} + \frac{1}{0.57490} \left[0.112128 \lambda^3 + 0.53524 \lambda^2 + 0.094670 \lambda + 0.154046 \right]$$

$$= 0.058164 + 0.431004 = \underline{\underline{0.4892}}$$

$$\left(\frac{\sigma R}{Et} \right) = \underline{0.4810} \quad (0.6905610)$$

$$\bar{E} = 0.2393166 + 64 \left[0.003660250 (-\lambda)^4 - 0.007320500 (-\lambda)^3 + 0.053014976 (-\lambda)^2 - 0.01154496 (-\lambda) + 0.010071909 \right] = \underline{\underline{0.85251}}$$

$$\Theta = + \underline{\underline{0.2418309}}$$

$$\Phi = + 0.022255$$

$$\boxed{\gamma = 0.144, \quad \xi = 8,}$$

$$175) = 1.152$$

$$175)^2 = 1.327104$$

656

$$1.0218701 \lambda^3 + 1.3471949 \lambda^2 - 2.6549149 \lambda + 0.1429250 = 0$$

$$F(\lambda) = \lambda^3 + 1.318362 \lambda^2 - 2.598094 \lambda + 0.1398661 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 2.636724 \lambda - 2.598094$$

$$F(0.054) = 0.0035708$$

$$F'(0.054) = 2.447$$

$$F(0.05546) = +0.0000014,$$

$$F'(0.05546) = 2.443$$

$$\lambda = -0.0554606, \quad \lambda^2 = +0.0030759, \quad \lambda^3 = -0.0001706$$

$$\frac{OR}{Et} = \frac{0.288}{1.73} \frac{1.8890788}{2.8870388} + \frac{1}{0.4160273} \left\{ 0.165868 \lambda^2 + 0.92565504 \lambda^2 + 0.1996506 \lambda \right. \\ \left. + 0.1120915 \right\}$$

$$= 0.068960 + 0.24959 = \underline{\underline{0.31857}}$$

$$\left(\frac{\xi R}{t} \right) = \underline{\underline{0.724}} \quad (0.51272(2))$$

$$\mathcal{E} = 0.1014818 + 64 \left\{ 0.005184000 (-\lambda)^6 - 0.010361070 (-\lambda)^3 + 0.062025601 (-\lambda)^4 \right. \\ \left. - 0.011891621 (-\lambda) + 0.008794132 \right\}$$

$$= \underline{\underline{0.636244}}$$

$$\Theta = \underline{\underline{+0.2221061}}$$

$$\Phi = +0.087624$$

$$\boxed{\gamma = 0.144, \quad \xi = 6,}$$

$$(\gamma\xi) = 0.864$$

157

$$(\gamma\xi)^2 = 0.746496$$

$$0.57480192 \lambda^3 + 1.2977971 \lambda^2 - 2.0466819 \lambda + 0.08703144 = 0$$

$$F(-\lambda) = \lambda^3 + 2.257817 \lambda^2 - 3.560674 \lambda + 0.1514112 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 4.515634 \lambda - 3.560674$$

$$F(0.0435) = +0.0008765$$

$$F'(0.0435) = -3.3586$$

$$F(0.043761) = 0.15$$

$$\lambda = -0.0437610, \quad \lambda^2 = +0.0019150, \quad \lambda^3 = -0.0000838$$

$$\frac{GR}{Et} = \frac{0.248}{2.73} \frac{1.912478}{2.912478} + \frac{1}{0.4193968} \left\{ 0.0933120 \lambda^3 + 0.5206410 \lambda^2 + 0.0610534 \lambda + 0.1886765 \right\}$$

$$= 0.06927 + 0.44586 = 0.51513$$

$$\left(\frac{\xi R}{t} \right) = \underline{\underline{6.7446}} \quad (0.5544292)$$

$$\bar{E} = 0.2653589 + 36 \left\{ 0.001911650 (-\lambda)^4 - 0.005212500 (-\lambda)^3 + 0.04962012 (-\lambda)^2 - 0.01275418 (-\lambda) + 0.011392521 \right\}$$

$$= \underline{\underline{0.6519661}}$$

$$\Theta = \underline{\underline{+0.1885496}}$$

$$\Phi = -0.054012$$

$$\boxed{\gamma = 0.169, \quad \xi = 5}$$

$$(\gamma\xi) = 0.845$$

$$(\gamma\xi)^2 = 0.714025$$

$$\frac{2\gamma}{3(1-\gamma)} = 0.1238095, \quad \frac{2\gamma^2}{3(1-\gamma)} = 0.0209238$$

$$0.54979925\lambda^3 + 1.2878011\lambda^2 - 2.0168828\lambda + 0.089648706 = 0$$

$$F(-\lambda) = \lambda^3 + 2.3423115\lambda^2 - 3.6684077\lambda + 0.1630572 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 4.6846230\lambda - 3.6684077$$

$$F(0.0457) = +0.0003913$$

$$F'(0.0457) = -3.4480$$

$$0.001155$$

$$F(0.0458155) = +0.0 -$$

$$\lambda = -0.0458155, \quad \lambda^2 = +0.0020991, \quad \lambda^3 = -0.0000962$$

$$\frac{\sigma R}{Et} = 0.1238095 \frac{1.9083690}{2.9583690} + \frac{1}{0.4915144} \left\{ 0.0892531\lambda^3 + 0.4980324\lambda^2 + 0.0569180\lambda + 0.1756721 \right\}$$

$$= 0.08124 + 0.39490 = \underline{\underline{0.47614}}$$

$$1.4390384$$

$$\left(\frac{\xi R}{t} \right) = \underline{\underline{0.6626}}$$

$$\gamma^2 = 0.0276$$

$$\begin{aligned} \xi &= 0.2217093 + 25 \left\{ 0.002789160(-\lambda)^4 - 0.105578320(-\lambda)^3 + 0.049424990(-\lambda)^2 \right. \\ &\quad \left. - 0.013632017(-\lambda) + 0.011954448 \right\} \end{aligned}$$

$$= \underline{\underline{0.5126233}}$$

$$\ominus = +0.1676036$$

$$\Phi = -0.059178$$

$$\boxed{\gamma = 0.169, \xi = 7.1}$$

$$1/\xi = 1.183$$

$$(\delta_3)^2 = 1.399489$$

659

$$1.07760653 \lambda^3 + 1.3410902 \lambda^2 - 2.299460 \lambda + 0.1556246 = 0$$

$$F(-\lambda) = \lambda^3 + 1.2445082 \lambda^2 - 2.5333421 \lambda + 0.1444169$$

$$F'(-\lambda) = 3\lambda^2 + 2.4890164 \lambda - 2.5333421$$

$$F(0.0590) = -0.0005128$$

$$\underline{0.002158}$$

$$F'(0.0590) = 2.376$$

$$F(0.058842) = 0.00$$

$$\lambda = -0.058842, \quad \lambda^2 = +0.0034556, \quad \lambda^3 = -0.002031$$

$$\frac{OR}{Et} = 0.1238095 \times \frac{1.1824316}{2.1124316} + \frac{1}{0.4871509} \{ 0.1749361 \lambda^3 + 0.9761436 \lambda^2 + 0.2230612 \lambda + 0.1071423 \}$$

$$= 0.05016 + 0.19988 = \underline{0.25004}$$

$$\left(\frac{\xi \lambda}{t} \right) = \underline{0.6403} \quad (0.4099261)$$

$$\lambda^2 = 0.0021561$$

$$\begin{aligned} \xi &= 0.0288149 + 4.0003466254 \lambda^2 - 0.010933508 (-\lambda)^3 + 0.064369763 (-\lambda)^2 \\ &- 0.012692272 (-\lambda) + 0.008724220 \end{aligned}$$

$$= \underline{0.490336}$$

$$\Theta = + \underline{0.1959878}$$

$$\Phi = + 0.065410$$

$$\boxed{\gamma = 0.1:9 \quad \xi = 9}$$

$$j\beta = 1.521$$

$$j\beta^2 = 1.313441$$

$$1.7813496 \lambda^3 + 1.1304756 \lambda^2 - 3.3991102 \lambda + 0.2435925 = 0$$

$$F(-\lambda) = \lambda^3 + 0.6346175 \lambda^2 - 1.9041657 \lambda + 0.1367410$$

$$F'(-\lambda) = 3\lambda^2 + 1.2692350 \lambda - 1.9041657$$

$$F(0.073) = +0.001221$$

$$F'(0.073) = 1.7995$$

$$F(0.0736716) = 0.00$$

$$\lambda = -0.0736716, \quad \lambda^2 = +0.0054215, \quad \lambda^3 = -0.004000$$

$$\frac{\sigma R}{Et} = 0.1234095 \times \frac{1.8526428}{2.8526428} + \frac{1}{0.4820966} \left\{ 0.4891801 \lambda^3 + 1.6136251 \lambda^2 + 0.5854175 \lambda + 0.0947275 \right\}$$

$$= 0.08041 + 0.12495 = \underline{\underline{0.20536}}$$

$$\left(\frac{\xi R}{t} \right) = \underline{\underline{0.7816}} \quad (0.6218905)$$

$$j^2 = 0.021561$$

$$\xi = 0.0421727 + 81 \left\{ 0.009036119 (-\lambda)^4 - 0.018073258 (\lambda)^3 + 0.084226101 (\lambda)^2 - 0.015140319 (-\lambda) + 0.007029150 \right\}$$

$$= \underline{\underline{0.5575996}}$$

$$\Phi = +0.11653$$

$$\Theta = -\underline{\underline{0.1033791}}$$

$$\boxed{\eta = 0.169 \quad \xi = 11} \quad \begin{aligned} (\eta\xi) &= 1.859 \\ (\eta\xi)^2 &= 3.455881 \end{aligned}$$

661

$$2.6610284\lambda^3 + 0.6559574\lambda^2 - 4.0240905\lambda + 0.3535523 = 0$$

$$F(-\lambda) = \lambda^3 + 0.2465052\lambda^2 - 1.5122315\lambda + 0.1328630 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 0.4930104\lambda - 1.5122315$$

$$F(0.0194) = +0.0003542, \quad F'(0.0194) = 1.4442$$

2453

$$F(0.0196453) = 0$$

$$\lambda = -0.0196453, \quad \lambda^2 = +0.0003863, \quad \lambda^3 = -0.0007204$$

$$\frac{\sigma_R}{Et} = 0.1238095 \times \frac{1.8207094}{2.8207094} + \frac{1}{0.4366199} \left\{ 0.4319151\lambda^3 + 2.4104710\lambda^2 + 1.1439221\lambda + 0.1584277 \right\}$$

$$= 0.07992 + 0.15219 = \underline{\underline{0.23211}}$$

$$\left(\frac{\sigma_R}{t} \right) = \underline{\underline{1.0914}} \quad (1.1911540) \quad \eta^2 = 0.02561$$

$$\bar{C} = 0.056212 + 121 \left\{ 0.013499535(-\lambda)^4 - 0.026999070(-\lambda)^3 + 0.10919212(-\lambda)^2 - 0.02304160(-\lambda) + 0.006424774 \right\}$$

$$= \underline{\underline{0.6872404}}$$

$$\Theta = -\underline{\underline{0.6230466}}$$

$$\Phi = +0.019838$$

$$\boxed{\beta = 0.225 \quad \xi = 4,}$$

$$(\gamma \xi) = 0.910$$

$$1/\xi^2 = 0.0610$$

6/2

$$\frac{2\beta}{3(1-\beta^2)} = 0.1648352,$$

$$\frac{2\beta^2}{3(1-\beta^2)} = 0.0370879$$

$$0.6237 \lambda^3 + 1.31445 \lambda^2 - 2.1682508 \lambda + 0.1150504 = 0$$

$$F(-\lambda) = \lambda^3 + 2.1075036 \lambda^2 - 3.4764322 \lambda + 0.1844643$$

$$F'(-\lambda) = 3\lambda^2 + 4.2150072 \lambda - 3.4764322$$

$$F(0.0549) = 0.0001257$$

$$F'(0.0549) = 3.2356$$

$$F(0.0549389) = 0.0000386$$

$$\lambda = -0.0549389, \quad \lambda^2 = +0.0030183, \quad \lambda^3 = -0.0001658$$

$$\frac{GR}{Et} = 0.1648352 \times \frac{1.8901222}{2.8901222} + \frac{1}{0.5503225} \left\{ 0.10125 \lambda^3 + 0.564975 \lambda^2 + 0.0705875 \lambda + 0.1760413 \right\}$$

$$= 0.10720 + 0.26741 = \underline{\underline{0.37521}}$$

$$(0.3733431)$$

$$\left(\frac{\varepsilon R}{t} \right) = \underline{\underline{0.5323}}$$

$$\beta^2 = 0.050625$$

$$\xi = 0.1407825 + 16 \left\{ 0.003164063 (-\lambda)^4 - 0.006324125 (-\lambda)^3 + 0.05354458 (-\lambda)^2 - 0.015221145 (-\lambda) + 0.012393484 \right\}$$

$$= \underline{\underline{0.328474}}$$

$$\ominus = + \underline{\underline{0.1585515}}$$

$$\Phi = -0.035590$$

$$\boxed{\gamma = 0.225 \quad \xi = 6}$$

$$(\xi) = 1.350$$

$$(\xi)^2 = 1.8225$$

63

$$1.4033250 \lambda^3 + 1.2760125 \lambda^2 - 3.0963446 \lambda + 0.21250353 = 0$$

$$F(-\lambda) = \lambda^3 + 0.9050024 \lambda^2 - 2.2064344 \lambda + 0.1514286 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 1.8100048 \lambda - 2.2064344$$

$$F(0.0707) = +0.0003107$$

$$F'(0.0707) = 2.06347$$

$$F(0.0707506) = 0.00$$

$$\lambda = -0.0707506, \quad \lambda^2 = +0.0050198, \quad \lambda^3 = -0.0003557$$

$$\frac{\gamma R}{Et} = 0.1648352 \times \frac{1.8582988}{2.8582988} + \frac{1}{0.6431172} \left\{ 0.2278125 \lambda^3 + 1.2711938 \lambda^2 + 0.3775719 \lambda + 0.0914953 \right\}$$

$$= 0.10717 + 0.11047 = \underline{0.21764}$$

$$\left(\frac{\gamma R}{Et} \right) = \underline{0.5640} \quad \left(\frac{\gamma R}{Et} \right) = 10.310960 \quad \gamma^2 = 0.05625$$

$$\gamma = 0.042322 + 36 \left\{ 0.007117141(-\lambda)^3 - 0.01423621(-\lambda)^2 + 0.02561029(-\lambda) - 0.015757278(-\lambda) + 0.008874566 \right\}$$

$$= \underline{0.3401417}$$

$$\ominus = +0.0693272$$

$$\Phi = +0.047226$$

$$\boxed{\gamma = 0.225, \quad \xi = 7.5}$$

$$175) = 1.6875$$

44

$$175)^2 = 2.84765625$$

$$2.1926953 \lambda^3 + 0.9297070 \lambda^2 - 3.7447520 \lambda + 0.3111748 = 0$$

$$F(-\lambda) = \lambda^3 + 0.4240019 \lambda^2 - 1.7078305 \lambda + 0.1419143 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 0.8480038 \lambda - 1.7078305$$

$$F(0.0852) = +0.0001035$$

$$F'(0.0852) = 1.6138$$

641

$$F(0.0852641) = 0.0.$$

$$\lambda = -0.0852641, \quad \lambda^2 = +0.0072700, \quad \lambda^3 = -0.0006199$$

$$\frac{\sigma R}{Et} = 0.1648352 \times \frac{1.6294718}{2.8294718} + \frac{1}{0.6366312} \left\{ 0.3559570 \lambda^3 + 1.9862402 \lambda^2 + 0.8360498 \lambda + 0.1165942 \right\}$$

$$= 0.10658 + 0.09351 = \underline{\underline{0.20009}}$$

(0.5353849)

$$\left(\frac{\xi f}{t} \right) = \underline{\underline{0.7317}}$$

$$\xi = 0.04036 + 5(25) \left\{ 0.011123(57(-\lambda)^4 - 0.022247314(-\lambda)^3 + 0.097955996(-\lambda)^2 - 0.020914370(-\lambda) + 0.007618746 \right\}$$

$$= \underline{\underline{0.4071002}}$$

$$\ominus = -0.2386782$$

$$\oplus = +0.057394$$

$$\boxed{\gamma = 0.225, \quad \xi = 7,}$$

$$(\gamma \xi) = 2025$$

$$(\gamma \xi)^2 = 4100625$$

$$3.1574813 \lambda^3 + 0.326281 \lambda^2 - 4.3473055 \lambda + 0.4917731 = 0$$

$$F(-\lambda) = \lambda^3 + 0.1033349 \lambda^2 - 1.3768269 \lambda + 0.1367460$$

$$F'(-\lambda) = 3\lambda^2 + 0.2066698 \lambda - 1.3768269$$

$$F(0.10) = +0.0010966$$

$$0.008270$$

$$F'(0.10) = -1.326$$

$$F(0.1008270) = 0.0$$

$$\lambda = -0.1008270, \quad \lambda^2 = +0.0101661, \quad \lambda^3 = -0.0010250$$

$$\frac{GR}{Et} = 0.1648352 \times \frac{1.7983460}{2.7983460} + \frac{1}{0.6296279} \left\{ 0.5125781 \lambda^3 + 2.8601859 \lambda^2 + 1.4901617 \lambda + 0.2475832 \right\}$$

$$= 0.10593 + 0.15229 = \underline{0.25822}$$

$$\left(\frac{ER}{t} \right) = \frac{1.0092}{(1.018446)} \quad \lambda^2 = 0.050625$$

$$\xi = 0.0661226 + 81 \left\{ 0.1601801 (-\lambda)^4 - 0.03201173 (-\lambda)^3 + 0.12500000 (-\lambda)^2 \right.$$

$$\left. - 0.03014717 (-\lambda) + 0.007544791 \right\}$$

$$= \underline{0.5325437}$$

$$\ominus = -0.4771215$$

$$\oplus = +0.05626$$

$$\boxed{\gamma = 0.420, \quad \xi = 1.}$$

$$(\gamma\xi) = 0.420$$

$$(\gamma\xi)^2 = 0.1600$$

666

$$\frac{\gamma^2}{3(1-\gamma^2)} = 0.2930403, \quad \frac{\gamma^2}{3(1-\gamma^2)} = 0.1172161$$

$$0.1232\lambda^3 + 0.8152\lambda^2 - 1.2036322\lambda + 0.1326161 = 0$$

$$F(-\lambda) = \lambda^3 + 6.6168631\lambda^2 - 9.7697419\lambda + 1.0264294 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 13.2337662\lambda - 9.7697419$$

$$F(0.120) = +0.0010715$$

$$F'(0.120) = 8.138$$

$$0.001317$$

$$F(0.1201317) = 0.00$$

$$\lambda = -0.1201317, \quad \lambda^2 = 0.0144316, \quad \lambda^3 = -0.0017337$$

$$\frac{\sigma R}{E t} = 0.2930403 \times \frac{1.2597366}{2.7597566} + \frac{1}{1.1038944} \{ 0.12\lambda^3 + 0.1116\lambda^2 + 0.1374\lambda + 0.4283 \}$$

$$= 0.18686 + 0.37446 = \underline{\underline{0.56132}}$$

$$\left(\frac{\varepsilon R}{t} \right) = \frac{0.5774}{(0.3335906)}$$

$$\gamma^2 = 0.16000$$

$$\xi = 0.3150801 + \left\{ 0.000625(-\lambda)^4 - 0.00125(-\lambda)^3 + 0.049369514(-\lambda)^2 \right. \\ \left. - 0.033139514(-\lambda) + 0.02348321 \right\}$$

$$= \underline{\underline{0.335594}} \quad \ominus = \underline{\underline{+0.0066085}}$$

$$\Phi = -0.156307$$

$$\boxed{\gamma = 0.490 \quad \xi = 2.5}$$

$$(\xi\gamma) = 1.000$$

$$(\xi\gamma)^2 = 1.000$$

6/2

$$0.77\lambda^3 + 1.345\lambda^2 - 2.5419322\lambda + 0.2134661 = 0$$

$$F(-\lambda) = \lambda^3 + 1.7467532\lambda^2 - 3.3012107\lambda + 0.2772247 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 3.4935064\lambda - 3.3012107$$

$$F(0.0883) = +0.0000395$$

$$0.000183$$

$$F'(0.0883) = 2.969$$

$$F(0.0883133) = 0.0$$

$$\lambda = -0.0883133, \quad \lambda^2 = +0.0077992, \quad \lambda^3 = -0.0006888$$

$$\frac{OR}{Et} = 0.2930403 \times \frac{1.8233734}{2.8233734} \times \frac{1}{1.129494} \left\{ 0.125\lambda^3 + 0.6975\lambda^2 + 0.10875\lambda + 0.145625 \right\}$$

$$= 0.18925 + 0.12518 = \underline{\underline{0.31443}}$$

$$\left(\frac{\Sigma R}{t} \right) = \underline{\underline{0.4190}} \quad (0.1755(10))$$

$$\xi = 0.0988662 + 6.25 \left\{ 0.0032625(-\lambda)^4 - 0.0076125(-\lambda)^3 + 0.067691129(-\lambda)^2 - 0.24230139(-\lambda) + 0.01637337 \right\}$$

$$= \underline{\underline{0.1912174}}$$

$$\Theta = +\underline{\underline{0.0891293}} \quad \Phi = -0.03409$$

$$\boxed{\gamma = 0.400 \quad \xi = 4}$$

$$1/\xi = 1.600$$

66f

$$1/\xi^2 = 2.560$$

$$1.9712 \lambda^3 + 1.0432 \lambda^2 - 3.7416332 \lambda + 0.3636161 = 0$$

$$F(-\lambda) = \lambda^3 + 0.5292208 \lambda^2 - 1.8921495 \lambda + 0.1844643 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 1.0584416 \lambda - 1.8921495 = 0$$

$$F(0.10) = +0.0009415,$$

$$F'(0.10) = 1.762$$

$$F(0.1005343) = 0, \text{ i.e.}$$

$$\lambda = -0.1005343, \quad \lambda^2 = +0.0101071, \quad \lambda^3 = -0.0010161$$

$$\frac{GR}{Et} = 0.2930403 \times \frac{1.7989314}{2.3989314} + \frac{1}{1.119574} \left\{ 0.32\lambda^3 + 1.7656\lambda^2 + 0.6984\lambda + 0.1026 \right\}$$

$$= 0.18834 + 0.04494 = \underline{\underline{0.23328}}$$

$$\left(\frac{\Sigma R}{t} \right) = \overset{(0.2471084)}{\underline{\underline{0.4921}}}$$

$$\begin{aligned} \bar{G} &= 0.0544196 + 16 \left\{ 0.01(-\lambda)^4 - 0.02(-\lambda)^3 + 0.101702014(-\lambda)^2 \right. \\ &\quad \left. - 0.029202014(-\lambda) + 0.012838510 \right\} \end{aligned}$$

$$= \underline{\underline{0.2290008}}$$

$$\Theta = \underline{\underline{-0.07327797}}$$

$$\Phi = -0.071459$$

$$\boxed{\gamma = 0.401, \quad \xi = 55}$$

$$(\xi)' = 2.20$$

$$(\xi)^2 = 4.84$$

669

$$3.7268\lambda^3 - 0.0902\lambda^2 - 48027322\lambda + 0.5830661 = 0$$

$$F(-\lambda) = \lambda^3 - 0.0242031\lambda^2 - 1.2117013\lambda + 0.1564522 = 0$$

$$F'(-\lambda) = 3\lambda^2 - 0.0484062\lambda - 1.2117013$$

$$F(0.10) = 0.043401$$

$$F'(0.10) = 1.2905$$

$$F(0.12195) = 0.0007487$$

$$F'(0.12195) = 1.2450$$

$$0.006014$$

$$F(0.1225494) = 0.0$$

$$\lambda = -0.1225494, \quad \lambda^2 = +0.0150184, \quad \lambda^3 = -0.0018405$$

$$\frac{QR}{Et} = 0.2930403 \times \frac{1.7549012}{2.7549012} + \frac{1}{1.1019605} \left\{ 0.605\lambda^3 + 3.3759\lambda^2 + 1.90135\lambda + 0.2991250 \right\}$$

$$= 0.18667 + 0.10508 = \underline{\underline{0.29175}}$$

$$(0.6046618)$$

$$\left(\frac{ER}{t} \right) = \underline{\underline{0.7776}}$$

$$\xi = 0.0851181 + 30.25 \left\{ 0.01890625(-\lambda)^4 - 0.03781250(-\lambda)^3 + 0.151395819(-\lambda)^2 - 0.046555139(-\lambda) + 0.012917340 \right\}$$

$$= \underline{\underline{0.3722048}}$$

$$\ominus = -\underline{\underline{0.3844414}}$$

$$\Phi = -0.040462$$

$$\boxed{\gamma = 0.526, \quad \xi = 0,} \quad \begin{array}{l} \xi \gamma = 0 \\ (\xi \gamma)' = 0 \end{array}$$

$$\frac{2\gamma}{3(1-\gamma)} = 0.4952381, \quad \frac{2\gamma^2}{3(1-\gamma^2)} = 0.3347810$$

$$\lambda = -0.5000500$$

$$\frac{\overline{\sigma R}}{Et} = 0.4952381 \times \frac{1}{2} + \frac{1}{1.352} \left\{ -0.25 + 0.25 \right\}$$

$$= 0.2476191 + 0.36912 = \underline{\underline{0.61744}}$$

$$\left(\frac{\overline{\sigma R}}{t} \right) = \underline{\underline{0.61744}}$$

$$\ominus = 0$$

$$\boxed{\gamma = 0.676 \quad \xi = 0.5}$$

$$(\gamma\xi) = 0.337$$

(7)

$$1/\xi = 0.114244$$

$$0.01796788\lambda^3 + 0.7130482\lambda^2 - 1.4925700\lambda + 0.3457770 = 0$$

$$F(-\lambda) = \lambda^3 + 8.1057791\lambda^2 - 16.9672158\lambda + 3.9307188 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 16.2115582\lambda - 16.9672158$$

$$F(0.266) = +0.0097930$$

$$F'(0.266) = -12.443$$

$$F(0.266460) = +0.0000186$$

$$F'(0.266460) = -12.428$$

$$\lambda = -0.2667875, \quad \lambda^2 = 0.0711756, \quad \lambda^3 = -0.0189118$$

$$\frac{OR}{Et} = 0.495238/x \frac{14664250}{2.4664250} + \frac{1}{1.6673033} \left\{ 0.0142405\lambda^3 + 0.0296852\lambda^2 + 0.1751020\lambda + 0.4711825 \right\}$$

$$= 0.29445 + 0.25774 = \underline{0.55219}$$

$$\left(\frac{\varepsilon_R}{t} \right) = \frac{10.311364}{0.5520}$$

$$\gamma^2 = 0.456926$$

$$\bar{\phi} = 0.041158 + 0.25 \left\{ 0.000426226(-\lambda)^2 - 0.000872531(-\lambda)^3 \right.$$

$$\left. + 0.045542211(-\lambda)^2 - 0.061931390(-\lambda) + 0.038354398 \right\}$$

$$= \underline{0.3117126}$$

$$\ominus = +0.00112$$

$$\bar{\phi} = -0.152211$$

$$\boxed{\gamma = 0.676, \quad \xi = 1, \quad 1/\xi = 0.676}$$

12

$$1/\xi^2 = 0.456976$$

$$0.3518715\lambda^3 + 1.1621927\lambda^2 - 2.2715941\lambda + 0.3787649 = 0$$

$$F(-\lambda) = \lambda^3 + 3.3028896\lambda^2 - 6.4557491\lambda + 1.0764296 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 6.6057792\lambda - 6.4557491$$

$$F(0.171) = 0.0740765 \quad F'(0.171) = 5.237$$

$$0.01414$$

$$F(0.18514) = 0.0007708 \quad F'(0.18514) = 5.1300$$

$$0.001503$$

$$F(0.1852903) = 0.0$$

$$\lambda = -0.1852903, \quad \lambda^2 = +0.0343325 \quad \lambda^3 = -0.0063615$$

$$\frac{OR}{Et} = 0.4952381 \times \frac{1.6294194}{2.6294194} + \frac{1}{1.7774115} \left\{ 0.0571220\lambda^3 + 0.3117408\lambda^2 + 0.0474441\lambda + 0.2684801 \right\}$$

$$= 0.30449 + 0.15205 = \underline{0.45654}$$

$$1/\xi^2 = 0.456976$$

$$(0.2344496)$$

$$\left(\frac{\varepsilon R}{t} \right) = \underline{0.4842}$$

$$\mathcal{E} = 0.2106259 + \left\{ 0.001285063(-\lambda)^4 - 0.003570125(-\lambda)^3 + 0.013054267(-\lambda)^2 - 0.054866955(-\lambda) + 0.033536773 \right\}$$

$$= \underline{0.24246}$$

$$\ominus = + \underline{0.03417}$$

$$\underline{\phi} = -0.100989$$

$$\underline{\eta = 0.676, \quad \xi = 0.75} \quad (\eta\xi) = 0.507$$

$$(\eta\xi)^2 = 0.257049$$

673

$$0.19792773 \lambda^3 + 0.9706084 \lambda^2 - 1.815801 \lambda + 0.3595220 = 0$$

$$F(-\lambda) = \lambda^3 + 4.9038526 \lambda^2 - 9.5367138 \lambda + 1.8164307 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 9.8077052 \lambda - 9.5367138$$

$$F(\underline{0.210}) = +0.0392417, \quad F'(0.210) = 7.3448$$

00534

$$F(\underline{0.21534}) = 0.0001764, \quad F'(0.21534) = 7.2656$$

000245

$$F(0.2153645) = 0. \kappa$$

$$\lambda = -0.2153645, \quad \lambda^2 = +0.0463819, \quad \lambda^3 = -0.0099890$$

$$\frac{\widehat{\Sigma R}}{Et} = 0.4952381 \times \frac{1.5692410}{2.5694710} + \frac{1}{1.7361272} \left\{ 0.0321311 \lambda^3 + 0.1792917 \lambda^2 \right. \\ \left. + 0.0869908 \lambda + 0.3603169 \right\}$$

$$= 0.30248 + 0.20127 = \underline{\underline{0.50375}}$$

$$\lambda^2 = 0.456976$$

$$(\underline{0.2179091})$$

$$\left(\frac{\widehat{\Sigma R}}{t} \right) = \underline{\underline{0.5176}}$$

$$\zeta = 0.2537641 + 0.5625 \left\{ 0.001004098 (-\lambda)^4 - 0.002008195 (-\lambda)^3 \right. \\ \left. + 0.078700483 (-\lambda)^2 - 0.057691698 (-\lambda) + 0.035796928 \right\}$$

$$= \underline{\underline{0.2689299}}$$

$$(-) = \underline{\underline{+0.0038076}}$$

$$\Phi = -0.126276$$

$$\boxed{\eta = 0.676, \quad \xi = 2.003}, \quad \eta\xi = 1.352$$

$$\eta\xi^2 = 1.827904$$

$$1.4074661\lambda^3 + 1.2687709\lambda^2 - 3.6976905\lambda + 0.5167168 = 0$$

$$F(-\lambda) = \lambda^3 + 0.9014447\lambda^2 - 2.6271596\lambda + 0.3628574 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 1.8028894\lambda - 2.6271596$$

$$F(0.144) = +0.0062248, \quad F'(0.144) = 2.3053$$

002700

$$F(0.14670) = 0.0000161, \quad F'(0.14670) = 2.298$$

0000044

$$\lambda = -0.1467044, \quad \lambda^2 = +0.0215222, \quad \lambda' = -0.0031574$$

$$\frac{GR}{Et} = 0.4952381 \times \frac{1.7065912}{2.7065912} + \frac{1}{1.8296557} \left\{ 0.2284880\lambda^3 + 1.2749630\lambda^2 \right. \\ \left. + 0.3797126\lambda + 0.0914205 \right\}$$

$$= 0.31226 + 0.03412 = \underline{\underline{0.34638}}$$

(0.0043040)

$$\frac{ER}{t} = \underline{\underline{0.04520}}$$

$$\eta^2 = 0.456976$$

$$\mathcal{E} = 0.1199791 + 4 \left\{ 0.007140250(\lambda)^4 - 0.014280500(-\lambda)^3 + 0.112940213(-\lambda)^2 \right. \\ \left. - 0.052987463(-\lambda) + 0.027419446 \right\}$$

$$= \underline{\underline{0.2083187}}$$

$$\odot = +0.01965$$

$$\Phi = -0.052404$$

$$\gamma = 0.626, \quad \xi = 3.500$$

$$(\gamma\xi) = 2.366$$

$$(\gamma\xi)^2 = 5.597956$$

125

$$4.3104261 \lambda^3 - 0.5506392 \lambda^2 - 5.5069555 \lambda + 0.8735843 = 0$$

$$F(-\lambda) = \lambda^3 - 0.1277459 \lambda^2 - 1.2725894 \lambda + 0.2041127$$

$$F'(-\lambda) = 3\lambda^2 - 0.2554918 \lambda - 1.2775894$$

$$F(0.155) = 0.0052961, \\ 0.004254$$

$$F'(0.155) = 1.2451$$

$$F(0.159254) = +0.0000056, \\ 0.000077$$

$$F'(0.159254) = 1.242$$

$$\lambda = -0.1592587, \quad \lambda^2 = +0.0253633, \quad \lambda^3 = -0.0040393$$

$$\frac{\tilde{R}}{Et} = 0.4952381 \times \frac{1.6814126}{2.6814126} + \frac{1}{1.8126822} \left\{ 0.6777445 \lambda^3 + 3.9075713 \lambda^2 + 2.3497647 \lambda + 0.3966941 \right\}$$

$$= 0.31055 + 0.06547 = \underline{\underline{0.37602}}$$

$$\left(\frac{\tilde{R}}{Et} \right) = \frac{(0.482110)}{0.6949}$$

$$\gamma^2 = 0.456976$$

$$\tilde{G} = 0.1413910 + 12.25 \left\{ 0.024667016 (-\lambda)^4 - 0.043737031 (-\lambda)^3 + 0.195115566 (-\lambda)^2 - 0.080626175 (-\lambda) + 0.027288177 \right\}$$

$$= \underline{\underline{0.2766157}} \quad \int \quad \ominus = -0.2200733 \quad \Xi = -0.012758$$

$$\varepsilon = + \left[\frac{\sigma}{E} + \frac{\gamma \lambda}{E} + \frac{1}{2} m^2 \left[\frac{1}{16} f_1^2 + \frac{1}{8} \left(\frac{1}{2} f_1 + f_2 \right)^2 \right] \right]$$

$$= (1-\nu^2) \frac{\sigma}{E} + (1+\nu) \frac{m^2}{2} \left[\frac{1}{16} f_1^2 + \frac{1}{32} f_1^2 + \frac{1}{8} f_1 f_2 + \frac{1}{8} f_2^2 \right]$$

$$= (1-\nu^2) \frac{\sigma}{E} + (1+\nu) \frac{m^2}{2} \left[\frac{3}{32} f_1^2 + \frac{1}{8} f_1 f_2 + \frac{1}{8} f_2^2 \right]$$

$$= (1-\nu^2) \frac{\sigma}{E} + (1+\nu) \frac{m^2}{16} f_1^2 \left[\frac{3}{4} + \lambda + \lambda^2 \right]$$

$$\frac{\varepsilon R}{t} = (1-\nu^2) \frac{\sigma R}{E t} + \frac{(1+\nu)}{16} \gamma \xi^2 \left[\lambda^2 + \lambda + \frac{3}{4} \right]$$

$$\boxed{\frac{\varepsilon R}{t} = 0.91 \frac{\sigma R}{E t} + 0.08125 \xi(1/\xi) \left[\lambda^2 + \lambda + 0.75000 \right]}$$

$$\boxed{\frac{\varepsilon R}{t} = \frac{\sigma R}{E t} + 0.0625 \xi(1/\xi) \left[\lambda^2 + \lambda + 0.75000 \right]}$$

$$\frac{w}{R} = f_0 + f_1 \left(\cos \frac{\pi x}{R} + \frac{1}{9} \cos \frac{3\pi x}{R} \right) \left(\cos \frac{\pi y}{R} - \frac{1}{4} \cos \frac{2\pi y}{R} \right) + f_2 \left(\cos \frac{2\pi x}{R} + \cos \frac{2\pi y}{R} \right) \quad \underline{\underline{67f}}$$

$$\frac{\partial w}{\partial x} = -m \left\{ f_1 \left(\sin \frac{\pi x}{R} + \frac{1}{3} \sin \frac{3\pi x}{R} \right) \left(\cos \frac{\pi y}{R} - \frac{1}{4} \cos \frac{2\pi y}{R} \right) + 2f_2 \sin \frac{2\pi x}{R} \right\}$$

$$\frac{\partial w}{\partial y} = -m \left\{ f_1 \left(\cos \frac{\pi x}{R} + \frac{1}{9} \cos \frac{3\pi x}{R} \right) \left(\sin \frac{\pi y}{R} - \frac{1}{2} \sin \frac{2\pi y}{R} \right) + 2f_2 \sin \frac{2\pi y}{R} \right\}$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x^2} = - \left(\frac{m}{R} \right)^2 \left\{ f_1 \left(\cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} \right) \left(\cos \frac{\pi y}{R} - \frac{1}{4} \cos \frac{2\pi y}{R} \right) + 4f_2 \cos \frac{2\pi x}{R} \right\}$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial y^2} = - \left(\frac{m}{R} \right)^2 \left\{ f_1 \left(\cos \frac{\pi x}{R} + \frac{1}{9} \cos \frac{3\pi x}{R} \right) \left(\cos \frac{\pi y}{R} - \cos \frac{2\pi y}{R} \right) + 4f_2 \cos \frac{2\pi y}{R} \right\}$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x \partial y} = \left(\frac{m}{R} \right)^2 \left\{ f_1 \left(\sin \frac{\pi x}{R} + \frac{1}{3} \sin \frac{3\pi x}{R} \right) \left(\sin \frac{\pi y}{R} - \frac{1}{2} \sin \frac{2\pi y}{R} \right) \right\}$$

$$\Delta \Delta F = E \left(\frac{m}{R} \right)^2 \left[m^2 f_1^2 \left\{ \left(\sin \frac{\pi y}{R} + \frac{1}{3} \sin \frac{3\pi y}{R} \right)^2 \left(\sin \frac{\pi x}{R} - \frac{1}{2} \sin \frac{2\pi x}{R} \right)^2 \right. \right. \quad \underline{\underline{I}}$$

$$\left. - \left(\cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} \right) \left(\cos \frac{\pi y}{R} + \frac{1}{9} \cos \frac{3\pi y}{R} \right) \left(\cos \frac{\pi x}{R} - \frac{1}{4} \cos \frac{2\pi x}{R} \right) \left(\cos \frac{\pi y}{R} - \cos \frac{2\pi y}{R} \right) \right\}$$

$$\begin{aligned} & - 2m^2 f_1 f_2 \left\{ \left(\cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} \right) \left(-\frac{1}{4} + \cos \frac{\pi y}{R} + \cos \frac{3\pi y}{R} - \frac{1}{4} \cos \frac{4\pi y}{R} \right) \right. \\ & \left. + \left(\frac{10}{9} \cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} + \frac{1}{9} \cos \frac{5\pi x}{R} \right) \left(\cos \frac{\pi y}{R} - \cos \frac{2\pi y}{R} \right) \right\} \quad \underline{\underline{II}} \end{aligned}$$

$$\begin{aligned} & - 16 f_2^2 \cos^2 \frac{2\pi x}{R} \cos \frac{2\pi y}{R} + \left\{ f_1 \left(\cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} \right) \left(\cos \frac{\pi y}{R} - \frac{1}{4} \cos \frac{2\pi y}{R} \right) + 4f_2 \cos \frac{2\pi x}{R} \right\} \\ & \quad \quad \quad \underline{\underline{III}} \end{aligned}$$

$$\begin{aligned}
I &= \left(\sin^2 \frac{\pi x}{R} + \frac{2}{3} \sin \frac{\pi x}{R} \sin \frac{3\pi x}{R} + \frac{1}{9} \sin^2 \frac{3\pi x}{R} \right) \left(\sin^2 \frac{\pi y}{R} - \sin \frac{\pi y}{R} \sin \frac{2\pi y}{R} + \frac{1}{4} \sin^2 \frac{2\pi y}{R} \right) \\
&\quad - \left(\cos^2 \frac{\pi x}{R} + \frac{10}{9} \cos \frac{\pi x}{R} \cos \frac{3\pi x}{R} + \frac{1}{9} \cos^2 \frac{3\pi x}{R} \right) \left(\cos^2 \frac{\pi y}{R} - \frac{5}{4} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} + \frac{1}{4} \cos^2 \frac{2\pi y}{R} \right) \\
&= \frac{1}{4} \left(\frac{10}{9} - \frac{1}{3} \cos \frac{2\pi x}{R} - \frac{2}{3} \cos \frac{4\pi x}{R} - \frac{1}{9} \cos \frac{6\pi x}{R} \right) \left(\frac{5}{4} - \cos \frac{\pi y}{R} - \cos \frac{2\pi y}{R} + \cos \frac{3\pi y}{R} - \frac{1}{4} \cos \frac{4\pi y}{R} \right) \\
&\quad - \frac{1}{4} \left(\frac{10}{9} + \frac{19}{9} \cos \frac{2\pi x}{R} + \frac{10}{9} \cos \frac{4\pi x}{R} + \frac{1}{9} \cos \frac{6\pi x}{R} \right) \left(\frac{5}{4} - \frac{5}{4} \cos \frac{\pi y}{R} + \cos \frac{2\pi y}{R} - \frac{5}{4} \cos \frac{3\pi y}{R} + \frac{1}{4} \cos \frac{4\pi y}{R} \right) \\
&= \frac{1}{4} \left\{ \frac{5}{4} \left(-\frac{11}{9} \cos \frac{2\pi x}{R} - \frac{16}{9} \cos \frac{4\pi x}{R} - \frac{8}{9} \cos \frac{6\pi x}{R} \right) \right. \\
&\quad + \frac{1}{4} \cos \frac{\pi y}{R} \left(\frac{10}{9} + \frac{107}{9} \cos \frac{2\pi x}{R} + \frac{24}{9} \cos \frac{4\pi x}{R} + \cos \frac{6\pi x}{R} \right) \\
&\quad - \cos \frac{2\pi y}{R} \left(\frac{20}{9} + \frac{16}{9} \cos \frac{2\pi x}{R} + \frac{4}{9} \cos \frac{4\pi x}{R} \right) \\
&\quad + \frac{1}{4} \cos \frac{3\pi y}{R} \left(10 + \frac{11}{9} \cos \frac{2\pi x}{R} + \frac{24}{9} \cos \frac{4\pi x}{R} + \frac{1}{9} \cos \frac{6\pi x}{R} \right) \\
&\quad \left. - \frac{1}{4} \cos \frac{4\pi y}{R} \left(\frac{20}{9} + \frac{16}{9} \cos \frac{2\pi x}{R} + \frac{4}{9} \cos \frac{4\pi x}{R} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
 I = - \left[\frac{110}{144} \cos \frac{2\pi x}{R} + \frac{40}{144} \cos \frac{4\pi x}{R} + \frac{10}{144} \cos \frac{6\pi x}{R} - \frac{10}{144} \cos \frac{8\pi x}{R} + \frac{20}{36} \cos \frac{10\pi x}{R} - \frac{10}{16} \cos \frac{12\pi x}{R} \right. \\
 + \frac{20}{144} \cos \frac{14\pi x}{R} - \frac{10}{144} \cos \frac{16\pi x}{R} + \frac{10}{144} \cos \frac{18\pi x}{R} - \frac{10}{144} \cos \frac{20\pi x}{R} + \frac{10}{144} \cos \frac{22\pi x}{R} - \frac{10}{144} \cos \frac{24\pi x}{R} \\
 + \frac{16}{36} \cos \frac{2\pi x}{R} + \frac{4}{36} \cos \frac{4\pi x}{R} - \frac{13}{144} \cos \frac{6\pi x}{R} - \frac{26}{144} \cos \frac{8\pi x}{R} - \frac{26}{144} \cos \frac{10\pi x}{R} + \frac{4}{36} \cos \frac{12\pi x}{R} \\
 \left. - \frac{4}{144} \cos \frac{14\pi x}{R} + \frac{16}{144} \cos \frac{16\pi x}{R} + \frac{16}{144} \cos \frac{18\pi x}{R} + \frac{16}{144} \cos \frac{20\pi x}{R} + \frac{16}{144} \cos \frac{22\pi x}{R} + \frac{16}{144} \cos \frac{24\pi x}{R} \right]
 \end{aligned}$$

$$\begin{aligned}
 II = - \left[-\frac{1}{4} \cos \frac{\pi x}{R} - \frac{1}{4} \cos \frac{3\pi x}{R} + \cos \frac{5\pi x}{R} + \frac{1}{4} \cos \frac{7\pi x}{R} + \frac{1}{4} \cos \frac{9\pi x}{R} + \frac{1}{4} \cos \frac{11\pi x}{R} \right. \\
 - \cos \frac{13\pi x}{R} + \cos \frac{15\pi x}{R} + \frac{1}{4} \cos \frac{17\pi x}{R} + \frac{1}{4} \cos \frac{19\pi x}{R} + \cos \frac{21\pi x}{R} + \cos \frac{23\pi x}{R} \\
 \left. - \frac{1}{4} \cos \frac{25\pi x}{R} + \cos \frac{27\pi x}{R} + \cos \frac{29\pi x}{R} \right]
 \end{aligned}$$

$$\begin{aligned}
 II = - \left[-\frac{1}{2} \cos \frac{\pi x}{R} - \frac{1}{2} \cos \frac{3\pi x}{R} + \frac{3}{4} \cos \frac{5\pi x}{R} + \frac{3}{4} \cos \frac{7\pi x}{R} + 4 \cos \frac{9\pi x}{R} + \frac{2}{9} \cos \frac{11\pi x}{R} \cos \frac{13\pi x}{R} \right. \\
 - \frac{20}{9} \cos \frac{15\pi x}{R} \cos \frac{17\pi x}{R} - 2 \cos \frac{19\pi x}{R} \cos \frac{21\pi x}{R} - \frac{9}{9} \cos \frac{23\pi x}{R} \cos \frac{25\pi x}{R} + 2 \cos \frac{27\pi x}{R} \cos \frac{29\pi x}{R} \\
 \left. - \frac{1}{2} \cos \frac{31\pi x}{R} \cos \frac{33\pi x}{R} - \frac{1}{2} \cos \frac{35\pi x}{R} \cos \frac{37\pi x}{R} \right]
 \end{aligned}$$

$$III = f_1 \left(c_0 \frac{m}{R} \cos \frac{m}{R} + c_0 \frac{3m}{R} \cos \frac{m}{R} - \frac{1}{4} c_0 \frac{m}{R} \cos \frac{2m}{R} - \frac{1}{4} c_0 \frac{3m}{R} \cos \frac{2m}{R} \right) + 4f_2 c_0 \frac{2m}{R}$$

$$\Delta \Delta F = -E \left(\frac{m}{R} \right)^2 \left[\frac{1}{16} c_0 \frac{2m}{R} \left(\frac{110}{144} f_1^2 m^2 - 4f_2 \right) + \frac{1}{256} c_0 \frac{4m}{R} \left(\frac{10}{144} f_1^2 m^2 \right) + \frac{1}{128} c_0 \frac{6m}{R} \left(\frac{10}{144} f_1^2 m^2 \right) - \frac{1}{4} c_0 \frac{m}{R} \left(\frac{10}{144} f_1^2 m^2 \right) \right]$$

$$+ \frac{1}{4} c_0 \frac{12m}{R} \left(\frac{20}{36} f_1^2 m^2 \right) - \frac{1}{4} c_0 \frac{3m}{R} \left(\frac{10}{16} f_1^2 m^2 \right) + \frac{1}{256} c_0 \frac{6m}{R} \left(\frac{20}{144} f_1^2 m^2 \right) - \frac{1}{256} c_0 \frac{2m}{R} \cos \frac{m}{R} \left(\frac{10}{144} f_1^2 m^2 \right)$$

$$- \frac{1}{256} c_0 \frac{4m}{R} \cos \frac{m}{R} \left(\frac{34}{144} f_1^2 m^2 \right) - \frac{1}{128} c_0 \frac{6m}{R} \cos \frac{m}{R} \left(\frac{1}{16} f_1^2 m^2 \right) + \frac{1}{128} c_0 \frac{2m}{R} \cos \frac{m}{R} \left(\frac{1}{16} f_1^2 m^2 \right) + \frac{1}{36} f_1^2 m^2 + 16 f_2^2 m^2$$

$$+ \frac{1}{128} c_0 \frac{4m}{R} \cos \frac{2m}{R} \left(\frac{4}{36} f_1^2 m^2 \right) - \frac{1}{128} c_0 \frac{2m}{R} \cos \frac{2m}{R} \left(\frac{13}{144} f_1^2 m^2 \right) - \frac{1}{128} c_0 \frac{4m}{R} \cos \frac{2m}{R} \left(\frac{26}{144} f_1^2 m^2 \right)$$

$$- \frac{1}{256} c_0 \frac{6m}{R} \cos \frac{3m}{R} \left(\frac{1}{144} f_1^2 m^2 \right) + \frac{1}{400} c_0 \frac{2m}{R} \cos \frac{4m}{R} \left(\frac{16}{144} f_1^2 m^2 \right) + \frac{1}{1024} c_0 \frac{4m}{R} \cos \frac{4m}{R} \left(\frac{1}{144} f_1^2 m^2 \right)$$

$$- c_0 \frac{m}{R} \left(\frac{1}{2} f_1 f_2 m^2 \right) - \frac{1}{4} c_0 \frac{3m}{R} \left(\frac{1}{2} f_1 f_2 m^2 \right) + \frac{1}{4} c_0 \frac{m}{R} \cos \frac{m}{R} \left(\frac{34}{9} f_1 f_2 m^2 - f_1 \right)$$

$$+ \frac{1}{128} c_0 \frac{3m}{R} \cos \frac{m}{R} \left(4 f_1 f_2 m^2 - f_1 \right) + \frac{1}{64} c_0 \frac{5m}{R} \cos \frac{m}{R} \left(\frac{2}{9} f_1 f_2 m^2 \right) - \frac{1}{128} c_0 \frac{m}{R} \cos \frac{2m}{R} \left(\frac{20}{9} f_1 f_2 m^2 - \frac{1}{4} f_1 \right)$$

$$- \frac{1}{128} c_0 \frac{3m}{R} \cos \frac{2m}{R} \left(2 f_1 f_2 m^2 - \frac{1}{4} f_1 \right) - \frac{1}{256} c_0 \frac{5m}{R} \cos \frac{2m}{R} \left(\frac{2}{9} f_1 f_2 m^2 \right) + \frac{1}{128} c_0 \frac{m}{R} \cos \frac{3m}{R} \left(-9 f_1 f_2 m^2 \right)$$

$$+ \frac{1}{256} c_0 \frac{3m}{R} \cos \frac{3m}{R} \left(-9 f_1 f_2 m^2 \right) - \frac{1}{128} c_0 \frac{m}{R} \cos \frac{4m}{R} \left(\frac{1}{2} f_1 f_2 m^2 \right) - \frac{1}{128} c_0 \frac{3m}{R} \cos \frac{4m}{R} \left(\frac{1}{2} f_1 f_2 m^2 \right) \left[\right]$$

$$\begin{aligned}
 \rho_1 = & \frac{2}{16} \left(\frac{110}{144} \sqrt{\rho_1^2 \rho_2^2 m^2} \right)^2 + \frac{2}{256} \left(\frac{144}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{2}{1296} \left(\frac{10}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{2}{16} \left(\frac{20}{36} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + 2 \left(\frac{10}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 \\
 & + \frac{2}{81} \left(\frac{10}{16} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{2}{256} \left(\frac{20}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{25} \left(\frac{10}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{289} \left(\frac{24}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{1369} \left(\frac{2}{16} \sqrt{\rho_1^2 \rho_2^2} \right)^2 \\
 & + \frac{1}{64} \left(\frac{16}{36} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{400} \left(\frac{4}{36} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{169} \left(\frac{83}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{625} \left(\frac{26}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 \\
 & + \frac{1}{2025} \left(\frac{1}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{400} \left(\frac{16}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{1024} \left(\frac{4}{144} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + 2 \left(\frac{1}{2} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{2}{81} \left(\frac{1}{2} \sqrt{\rho_1^2 \rho_2^2} \right)^2 \\
 & + \frac{1}{4} \left(\frac{1}{4} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{100} \left(\frac{1}{4} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{676} \left(\frac{1}{9} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{25} \left(\frac{1}{4} \sqrt{\rho_1^2 \rho_2^2} \right)^2 \\
 & + \frac{1}{169} \left(\frac{1}{2} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{841} \left(\frac{1}{9} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{100} \left(\frac{1}{2} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{324} \left(\frac{1}{2} \sqrt{\rho_1^2 \rho_2^2} \right)^2 + \frac{1}{289} \left(\frac{1}{2} \sqrt{\rho_1^2 \rho_2^2} \right)^2 \\
 & + \frac{1}{625} \left(\frac{1}{2} \sqrt{\rho_1^2 \rho_2^2} \right)^2
 \end{aligned}$$

$$\begin{aligned}
 \rho_1 = & \left(\rho_1^2 \rho_2^2 \right)^2 \left\{ \frac{24200}{16} + \frac{12800}{256} + \frac{200}{1296} + 800 + 200 + 200 + \frac{800}{256} + \frac{11449}{25} + \frac{5436}{289} + \frac{81}{1369} \right. \\
 & + 64 + \frac{16}{25} + \frac{6889}{169} + \frac{626}{625} + \frac{1}{1025} + \frac{256}{400} + \frac{16}{1024} \left. \right\} \\
 & + \rho_1^2 \rho_2^2 m^4 \left\{ \frac{8}{36} + \frac{1}{2} + \frac{1}{162} + \frac{361}{81} + \frac{16}{100} + \frac{1}{13689} + \frac{16}{81} + \frac{4}{169} + \frac{4}{68121} + \frac{1}{25} + \frac{1}{81} + \frac{1}{1152} \right. \\
 & + \frac{1}{2500} \left. \right\} + 4 \left(\rho_1^2 \rho_2^2 \right)^2 - \left\{ \frac{110}{144} \rho_1^2 \rho_2^2 m^2 + \frac{18}{9} \rho_1^2 \rho_2^2 m^4 + \frac{2}{25} \rho_1^2 \rho_2^2 m^2 + \frac{2}{45} \rho_1^2 \rho_2^2 m^2 + \frac{1}{169} \rho_1^2 \rho_2^2 m^2 \right\} \\
 & + \left\{ 2 \rho_1^2 + \frac{1}{4} \rho_1^2 + \frac{1}{100} \rho_1^2 + \frac{1}{400} \rho_1^2 + \frac{1}{2704} \rho_1^2 \right\}
 \end{aligned}$$

68

$$\rho_1 = 0.16154936 f_1^4 m^4 + 5.6200987 f_1^2 f_2^2 m^4 + 4 f_2^4 m^4 - 3.0053616 f_1^2 f_2^2 m^2 + 0.26286982 f_1^2 + 2 f_2^2$$

$$\begin{aligned} \frac{1}{R} \frac{\partial^2 \rho}{\partial x^2} &= -\left(\frac{m}{R}\right)^2 \left\{ f_1 \left(\cos \frac{m x}{R} \cos \frac{m y}{R} + \cos \frac{3 m x}{R} \cos \frac{m y}{R} - \frac{1}{4} \cos \frac{m x}{R} \cos \frac{3 m y}{R} - \frac{1}{4} \cos \frac{3 m x}{R} \cos \frac{m y}{R} \right) + 4 f_2 \cos \frac{2 m x}{R} \right\} \\ \frac{1}{R} \frac{\partial^2 \rho}{\partial y^2} &= -\left(\frac{m}{R}\right)^2 \left\{ f_1 \left(\cos \frac{m x}{R} \cos \frac{m y}{R} + \frac{1}{9} \cos \frac{3 m x}{R} \cos \frac{m y}{R} - \cos \frac{m x}{R} \cos \frac{7 m y}{R} - \frac{1}{9} \cos \frac{3 m x}{R} \cos \frac{5 m y}{R} \right) + 4 f_2 \cos \frac{2 m y}{R} \right\} \\ \frac{1}{R} \frac{\partial^2 \rho}{\partial x \partial y} &= \left(\frac{m}{R}\right)^2 \left\{ f_1 \left(\sin \frac{m x}{R} \sin \frac{m y}{R} + \frac{1}{3} \sin \frac{3 m x}{R} \sin \frac{m y}{R} - \frac{1}{2} \cos \frac{m x}{R} \sin \frac{2 m y}{R} - \frac{1}{6} \sin \frac{3 m x}{R} \sin \frac{2 m y}{R} \right) \right\} \end{aligned}$$

$$\rho_2 = \frac{1}{12(1-\nu)} \left(\frac{1}{R}\right)^2 m^4 \left\{ 4 f_1^2 + \frac{100}{81} f_1^2 + \frac{25}{16} f_1^2 + \frac{169}{1296} f_2^2 + 64 f_2^2 \right\}$$

$$\rho_2 = \frac{1}{12(1-\nu)} \left(\frac{1}{R}\right)^2 m^4 \left\{ 6.927469 f_1^2 + 64 f_2^2 \right\}$$

$$\frac{\partial^2 \rho}{\partial x^2} = -m^2 \left\{ f_1 \left(\cos \frac{m x}{R} \sin \frac{m y}{R} + \frac{1}{9} \cos \frac{3 m x}{R} \sin \frac{m y}{R} - \frac{1}{2} \cos \frac{m x}{R} \sin \frac{2 m y}{R} - \frac{1}{18} \cos \frac{3 m x}{R} \sin \frac{2 m y}{R} \right) + 2 f_2 \sin \frac{2 m y}{R} \right\}$$

$$\frac{\Delta}{E} + 4 \frac{\sigma}{E} - \frac{1}{2} m^2 \left\{ f_1^2 \left(\frac{1}{4} + \frac{1}{324} + \frac{1}{16} + \frac{1}{1296} \right) + 2 f_2^2 \right\} + f_0 = 0$$

$$\boxed{\frac{1}{E} = m^2 \left\{ \frac{205}{1296} f_1^2 + f_2^2 \right\} - f_0 - 4 \frac{E}{E}}$$

$$\frac{\partial \mathcal{L}}{\partial x} = -m^2 \left\{ f_1 \left(\sin \frac{m x}{R} \cos \frac{m x}{R} + \frac{1}{3} \sin \frac{3 m x}{R} \cos \frac{m x}{R} - \frac{1}{4} \sin \frac{2 m x}{R} \cos \frac{2 m x}{R} - \frac{1}{12} \sin \frac{3 m x}{R} \cos \frac{2 m x}{R} \right) + 2 f_2 \sin \frac{2 m x}{R} \right\}$$

$$8 \mathcal{L}_0 = -8 \frac{E}{E} \left[\left(\frac{\sigma}{E} + \nu \frac{1}{E} \right) + \frac{1}{2} m^2 \left\{ f_1^2 \left(\frac{1}{4} + \frac{1}{36} + \frac{1}{64} + \frac{1}{576} \right) + 2 f_2^2 \right\} \right]$$

$$= -8 \frac{E}{E} \left[\left(\frac{\sigma}{E} + \nu \frac{1}{E} \right) + m^2 \left\{ \frac{85}{576} f_1^2 + f_2^2 \right\} \right]$$

$$= -8 \frac{E}{E} \left[(1-\nu^2) \left(\frac{\sigma}{E} \right) + m^2 \left\{ \left(\frac{85}{576} + \nu \frac{205}{1296} \right) f_1^2 + (1+\nu) f_2^2 \right\} - 4 f_0 \right]$$

$$\mathcal{L}_0 = -4 \left[2(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + m^2 \left\{ \left(\frac{15}{576} + \nu \frac{205}{648} \right) f_1^2 + 2(1+\nu) f_2^2 \right\} - 2 \nu f_0 \right]$$

$$4 \left[\left(\frac{\sigma}{E} + \nu \frac{1}{E} \right)^2 + 2 \nu \frac{\sigma}{E} \left(\frac{\sigma}{E} \right) - 4 \right] = 4 \left[(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + m^2 \left\{ \frac{205}{1296} f_1^2 + f_2^2 \right\} + f_0^2 - 2 f_0 f_0 \right] + \frac{205}{1296} f_1^2 + f_2^2$$

$$= 2 \nu \frac{\sigma}{E} \left[\frac{205}{1296} f_1^2 + 2 f_2^2 \right] + \left\{ \frac{205}{1296} f_1^2 + 2 f_2^2 \right\} + 2 \nu \frac{\sigma}{E} + \frac{E}{E} \frac{\sigma}{E} - 2 \nu f_0 \frac{E}{E}$$

684

$$K = -4 \left[(1-\nu) \left(\frac{E}{\sigma} \right)^2 + \left(\frac{E}{\sigma} \right) u + \left(\frac{E}{\sigma} \right) w + \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 \right] - \frac{3}{2} u + \frac{3}{2} w + \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 + \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 - \frac{3}{2} \left(\frac{E}{\sigma} \right)^2$$

$$\left[\left\{ 3f_2^2 + 4f_1^2 + \frac{849}{502} f_2^2 + 2f_1^2 \right\} w^2 + f_0^2 + f_2^2 - \left\{ 2f_2 + 4f_1 + \frac{961}{1296} \right\} w - \right.$$

$$\left. 24 \frac{E}{\sigma} + 2f_0 + \frac{205}{502} f_2^2 + 2f_1^2 \right\} = 0$$

\therefore

$$K = -4 \left[(1-\nu) \left(\frac{E}{\sigma} \right)^2 + \left(\frac{E}{\sigma} \right) u + \left(\frac{E}{\sigma} \right) w + \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 + \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 + \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 - \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 \right]$$

$$K = -4 \left(\frac{E}{\sigma} \right)^2 - \frac{3}{2} \left(\frac{E}{\sigma} \right)^2 - \frac{3}{2} \left(\frac{E}{\sigma} \right)^2$$

$$\frac{85}{36} \frac{\sigma_R}{E_t} \gamma = (\gamma \xi)^2 \left(112401974 \gamma^2 + 0.64619244 \right) - (\gamma \xi) (6.0107232 \gamma) + 0.52573964 + \frac{1}{6(1-\nu^2)} \gamma^2 6.9274891$$

$$16 \frac{\sigma_R}{E_t} \gamma \rho = (\gamma \xi)^2 \left(16 \rho^3 + 111401974 \gamma \right) - (\gamma \xi) (3.0053616) + 4 \gamma + \frac{1}{6(1-\nu^2)} \gamma^2 64 \gamma$$

$$\frac{\sigma_R}{E_t} \gamma = (\gamma \xi)^2 \left(4.7605542 \gamma^2 + 0 + 0.27368362 \right) - (\gamma \xi) (2.5457181 \gamma) + 0.22266620 + \frac{\gamma^2}{6(1-\nu^2)} 2.9339869$$

$$\frac{\sigma_R}{E_t} \gamma \rho = (\gamma \xi)^2 \left(\gamma^3 + 0.70251414 \gamma + 0 \right) - (\gamma \xi) (0.18783558) + 0.25 \gamma + \frac{\gamma^2}{6(1-\nu^2)} 4 \gamma$$

$$(\gamma \xi)^2 \left(3.7605542 \gamma^3 - 0.42883052 \gamma \right) - (\gamma \xi) (2.5457181 \gamma^2 - 0.18783558) - 0.027333380 \gamma$$

$$= - \frac{\gamma^2}{6(1-\nu^2)} 1.0660131 \gamma = 0$$

$$\gamma_{lim} \left\{ 3.7605542 (\gamma \xi)^2 \right\} \gamma^3 - \left\{ 2.5457181 (\gamma \xi) \right\} \gamma^2 - \left\{ 0.42883052 (\gamma \xi)^2 + 0.027333380 + 0.19524049 \gamma^2 \right\} \gamma$$

$$+ 0.18783558 (\gamma \xi) = 0$$

$$(175)^2 \phi^3 - 0.6769529(175)\phi^2 - \left(0.1140339(175)^2 + 0.05191801\phi^2 + 0.007268556\phi \right) + 0.04996891(175) = 0$$

$$\frac{dR}{dt} = \frac{1}{Y} \left\{ (175)^2 (4.7605542\phi^2 + 0.47368362) - 1.5457181(175)\phi + 0.22166620\phi + 0.5343602\phi \right\}$$

$$\left(\frac{dR}{dt} \right)_{\substack{\text{at } x=0 \\ Y=0}} = f_0 + f_1 \left(\frac{12}{4} \times \frac{3}{4} \right) + f_2(2)$$

$$\left(\frac{dR}{dt} \right)_{\substack{x=0 \\ Y=\frac{R}{2}}} = f_0 - f_1 \left(\frac{10}{9} \times \frac{7}{4} \right) + f_2(0)$$

$$\text{Amplitude} = \frac{30+50}{36} f_1 = \frac{80}{36} f_1 = \frac{20}{9} f_1$$

$$\boxed{\gamma = 0.100 \quad \xi = (16 \times \frac{9}{20}) = 7.2}$$

68

$$(\gamma\xi) = 0.72, \quad (\gamma\xi)^2 = 0.5184$$

$$0.5184 \xi^3 - 0.4874061 \xi^2 - 0.06690291 \xi + 0.03596322 = 0$$

$$F(\xi) = \xi^3 - 0.9402124 \xi^2 - 0.1290565 \xi + 0.0693735 = 0$$

$$F'(\xi) = 3\xi^2 - 1.8804248 \xi - 0.1290565$$

$$F(0.235) = +0.0000999, \quad F'(0.235) = 0.40528$$

0.00246

$$F(0.235246) = \text{O.K.}, \quad \xi = +0.235246, \quad \xi^2 = 0.0553407$$

$$\frac{\sigma_R}{Et} = 10 \left\{ 0.5375645 (\gamma\xi)^2 - 0.5988700 (\gamma\xi) + 0.22266620 \right\} + 0.05373602$$

$$= \underline{\underline{0.75527}}$$

$$\boxed{\gamma = 0.081 \quad \xi = 7.2}$$

$$\gamma^2 = 0.006561$$

689

$$(\gamma\xi) = 0.5832, \quad (\gamma\xi)^2 = 0.34012224$$

$$0.34012224 \xi^3 - 0.3947989 \xi^2 - 0.04639466 \xi + 0.02913020 = 0$$

$$F(\xi) = \xi^3 - 1.1607559 \xi^2 - 0.1364058 \xi + 0.0756463 = 0$$

$$F'(\xi) = 3\xi^2 - 2.3215118 \xi - 0.1364058$$

$$F(0.239) = 0.0003937$$

$$0.007573$$

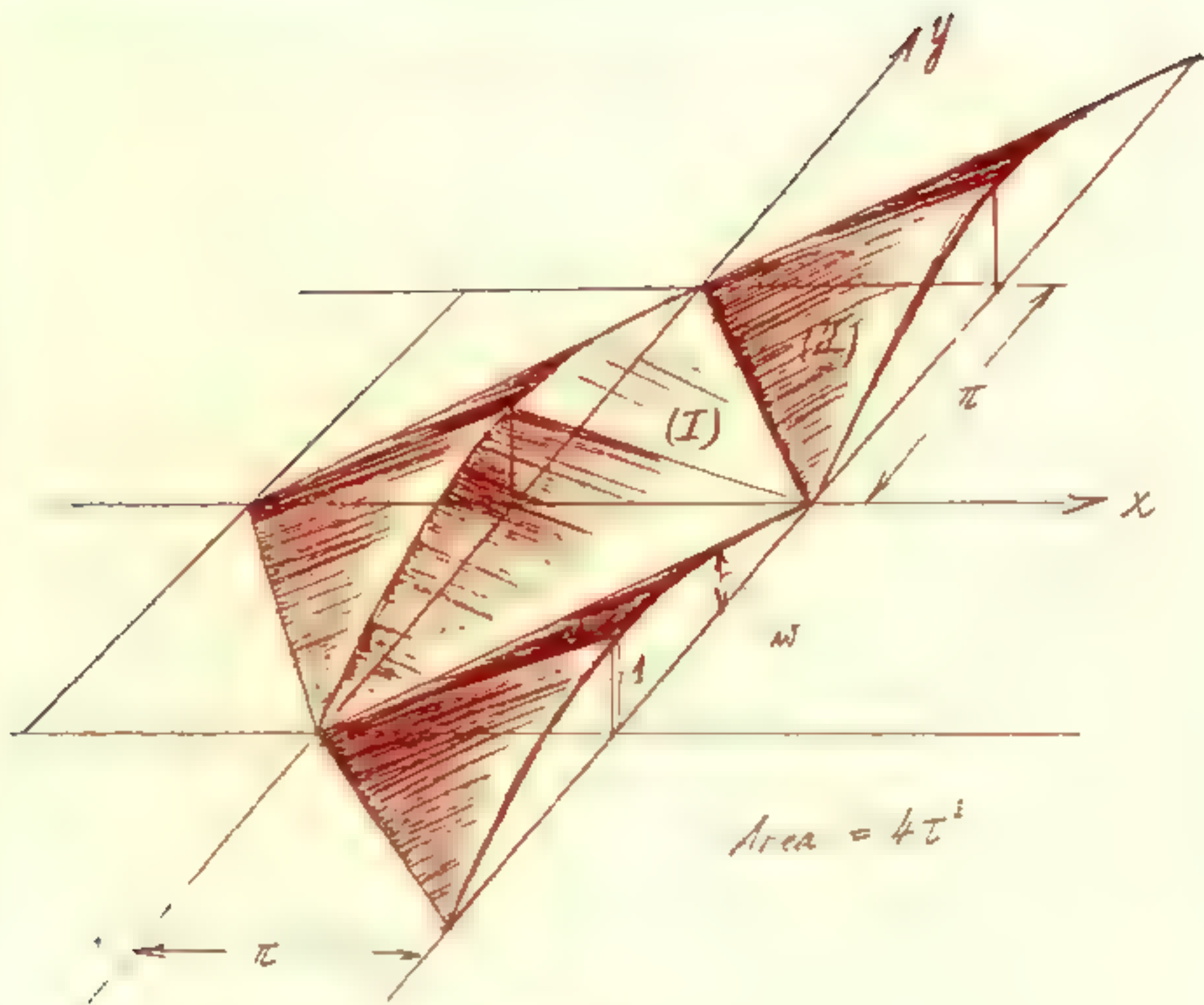
$$F'(0.239) = 0.51988$$

$$F(0.2392573) = 0.0000000$$

$$\xi = +0.2392573, \quad \xi^2 = 0.05746356$$

$$\frac{\sigma_R}{Et} = \frac{1}{0.081} \{ 0.5473372 (\gamma\xi)^2 - 0.6101545 (\gamma\xi) + 0.22266620 \} + 0.043762$$

$$= \underline{\underline{0.69623}}$$



$$w = \left[1 - \left(\frac{y}{\pi} \right)^2 \right] \left[1 - \frac{(x/\pi)}{1 - (y/\pi)^2} \right]$$

$$= \left[1 - \left(\frac{y}{\pi} \right)^2 \right] - (1 + \frac{y}{\pi}) \left(\frac{x}{\pi} \right) \quad \text{for (I)}$$

$$w = \left[1 - \left(1 - \frac{y}{\pi} \right)^2 \right] \frac{(\frac{x}{\pi}) - (1 - \frac{y}{\pi})}{(\frac{y}{\pi})}$$

$$= (2 - \frac{y}{\pi}) \left(\frac{x}{\pi} + \frac{y}{\pi} - 1 \right) \quad \text{for (II)}$$

$$w = a_{00} + a_{10} \cos x + a_{20} \cos 2x + a_{30} \cos 3x + a_{01} \cos y + a_{02} \cos 2y + a_{03} \cos 3y + a_{11} \cos x \cos y + a_{12} \cos x \cos 2y + a_{13} \cos x \cos 3y + \dots$$

$$\begin{aligned}
 a_{10} &= \frac{8}{4\pi^2} \int_0^\pi \int_0^{\pi-x} \left[\left\{ 1 - \left(\frac{y}{\pi} \right)^2 \right\} - \left(\frac{x}{\pi} \right) \left(1 + \frac{y}{\pi} \right) \right] dy dx \\
 &= \frac{2}{\pi^2} \int_0^\pi \left[\left\{ 1 - \frac{4}{3} \left(\frac{y}{\pi} \right)^2 \right\} - \left(\frac{x}{\pi} \right) \left(1 + \frac{1}{2} \frac{y}{\pi} \right) \right]_{y=0}^{\pi-x} dx \\
 &= \frac{2}{\pi} \int_0^\pi \left[\left(1 - \frac{x}{\pi} \right) \left\{ 1 - \frac{4}{3} \left(1 - \frac{x}{\pi} \right)^2 \right\} - \left(\frac{x}{\pi} \right) \left(1 - \frac{x}{\pi} \right) \left(1 + \frac{1}{2} \left(1 - \frac{x}{\pi} \right) \right) \right] dx \\
 &= 2 \int_0^1 \left[\eta \left\{ 1 - \frac{4}{3} \eta^2 \right\} + \eta^2 \left(1 + \frac{1}{2} \eta \right) - \eta \left(1 + \frac{1}{2} \eta \right) \right] d\eta \\
 &= 2 \int_0^1 \left(\eta - \frac{4}{3} \eta^3 + \eta^2 + \frac{1}{2} \eta^3 - \eta - \frac{1}{2} \eta^2 \right) d\eta \\
 &= 2 \int_0^1 \left(\frac{1}{2} \eta^2 + \frac{1}{6} \eta^3 \right) d\eta = 2 \left[\frac{1}{6} + \frac{1}{24} \right] = \frac{5}{12} = \underline{\underline{\frac{5}{12}}}
 \end{aligned}$$

$$\begin{aligned}
 a_{10} &= \frac{2}{\pi^2} \left[\int_0^\pi \int_0^{\pi-x} \left[\left\{ 1 - \left(\frac{y}{\pi} \right)^2 \right\} - \left(\frac{x}{\pi} \right) \left(1 + \frac{y}{\pi} \right) \right] \cos x dy dx \right. \\
 &\quad \left. + \int_0^\pi \int_{\pi-x}^\pi \left(2 - \frac{y}{\pi} \right) \left(\frac{1}{\pi} + \frac{y}{\pi} - 1 \right) \cos x dy dx \right] \\
 &= \frac{2}{\pi} \left\{ \int_0^\pi \left[\frac{1}{2} \left(1 - \frac{x}{\pi} \right)^2 + \frac{1}{6} \left(1 - \frac{x}{\pi} \right)^3 \right] \cos x dx \right. \\
 &\quad \left. + \int_0^\pi \left[\frac{2}{3} - \frac{3}{2} \left(1 - \frac{y}{\pi} \right) + \left(1 - \frac{y}{\pi} \right)^2 - \frac{1}{6} \left(1 - \frac{y}{\pi} \right)^3 \right] \cos x dx \right\} \\
 &= \frac{2}{\pi} \int_0^\pi \left\{ \frac{2}{3} - \frac{3}{2} \left(1 - \frac{y}{\pi} \right) + \frac{3}{2} \left(1 - \frac{y}{\pi} \right)^2 \right\} \cos x dx
 \end{aligned}$$

$$\text{Now } \int_0^\pi \left(1 - \frac{x}{\pi}\right)^n \cos x \, dx = \pi \int_0^1 (1-\xi)^n \cos \pi \xi \, d\xi$$

$$= -\pi \int_0^1 \eta^n \cos \eta \pi \, d\eta = -\frac{1}{\pi^n} \int_0^\pi \xi^n \cos \xi \, d\xi$$

$$a_{10} = -2 \left\{ \frac{1}{2\pi^3} \cdot (-2\pi) + \frac{1}{6\pi^4} \left(-\cancel{(3\pi^2-6)} + 6 \right) - \frac{3}{2\pi^2} (-1-1) \right. \\ \left. + \frac{1}{\pi^3} (-2\pi) - \frac{1}{6\pi^4} \left(-\cancel{(3\pi^2-6)} + 6 \right) \right\}$$

$$= -2 \left\{ -\frac{3}{\pi^2} + \frac{3}{\pi^2} \right\} = \underline{\underline{0}} = a_{10}$$

$$\int_0^\pi \left(1 - \frac{x}{\pi}\right)^n \cos 2x \, dx = + \frac{\pi}{(2\pi)^{n+1}} \int_0^{2\pi} \xi^n \cos \xi \, d\xi$$

$$a_{20} = +2 \left\{ -\frac{3}{2} \frac{1}{4\pi^3} (0) + \frac{3}{2} \frac{1}{4\pi^3} (4\pi) \right\} = \underline{\underline{+\frac{3}{2\pi^2}}} = a_{20}$$

$$a_{30} = -2 \left\{ -\frac{3}{2} \frac{1}{9\pi^2} (-1-1) + \frac{3}{2} \frac{1}{34\pi^3} (-6\pi) \right\}$$

$$= -2 \left\{ \frac{1}{3\pi^2} - \frac{1}{3\pi^2} \right\} = \underline{\underline{0}} = a_{30}$$

$$\begin{aligned}
a_{01} &= \frac{2}{\pi^2} \left[\int_0^\pi \int_0^{\pi-y} \left[\left\{ 1 - \left(\frac{x}{\pi} \right)^2 \right\} - \left(\frac{x}{\pi} \right) \left(1 + \frac{x}{\pi} \right) \right] dx \cos y dy \right. \\
&\quad \left. + \int_0^\pi \int_{\pi-y}^\pi \left[2 \left(\frac{x}{\pi} - 1 + \frac{x}{\pi} \right) - \frac{x}{\pi} \left(\frac{x}{\pi} - 1 + \frac{x}{\pi} \right) \right] dx \cos y dy \right] \\
&= \frac{2}{\pi} \left[\int_0^\pi \left[\left(1 - \frac{x}{\pi} \right) \left(1 - \left(\frac{x}{\pi} \right)^2 \right) - \frac{x}{\pi} \left(1 + \frac{x}{\pi} \right) \left(1 - \frac{x}{\pi} \right)^2 \right] \cos y dy \right. \\
&\quad \left. + \int_0^\pi \left[2 \left(\frac{x}{2} - 1 + \frac{x}{\pi} \right) - \frac{x}{\pi} \left(\frac{x}{2} - 1 + \frac{x}{\pi} \right) - 2 \left(1 - \frac{x}{\pi} \right) \left(\frac{x}{2} \left(1 - \frac{x}{\pi} \right) - 1 + \frac{x}{\pi} \right) \right. \right. \\
&\quad \left. \left. + \left(1 - \frac{x}{\pi} \right) \frac{x}{\pi} \left(\frac{x}{2} \left(1 - \frac{x}{\pi} \right) - 1 + \frac{x}{\pi} \right) \right] \cos y dy \right] \\
&= \frac{2}{\pi} \int_0^\pi \cos y \left[\frac{x}{2} \left(1 - \frac{x}{\pi} \right) \left(1 - \left(\frac{x}{\pi} \right)^2 \right) + \left(2 - \frac{x}{\pi} \right) \left(\frac{x}{\pi} - \frac{x}{2} \right) \right. \\
&\quad \left. + \frac{1}{2} \left(2 - \frac{x}{\pi} \right) \left(1 - \frac{x}{\pi} \right) \left(1 - \frac{x}{\pi} \right) \right] dy \\
&= \frac{2}{\pi} \int_0^\pi \cos y \left[\frac{x}{2} - \frac{x^2}{2\pi} - \frac{x}{2} \left(\frac{x}{\pi} \right)^2 + \frac{x}{2} \left(\frac{x}{\pi} \right)^3 + 2 \left(\frac{x}{\pi} \right) - \frac{x^2}{\pi} - 1 + \frac{x}{\pi} \right. \\
&\quad \left. + 1 - 2 \frac{x}{\pi} + \left(\frac{x}{\pi} \right)^2 - \frac{x}{2} \left(\frac{x}{\pi} \right) + \left(\frac{x}{\pi} \right)^2 - \frac{x}{2} \left(\frac{x}{\pi} \right)^3 \right] dy \\
&= \frac{2}{\pi} \int_0^\pi \left[\frac{x}{2} - \frac{x}{2} \left(\frac{x}{\pi} \right) + \frac{x}{2} \left(\frac{x}{\pi} \right)^3 \right] \cos y dy \\
&= \frac{1}{\pi} \int_0^\pi \left[1 - \left(\frac{x}{\pi} \right) + \left(\frac{x}{\pi} \right)^3 \right] \cos y dy
\end{aligned}$$

$$a_{01} = \frac{1}{\pi} \left[-\frac{1}{\pi}(-1-1) + \frac{1}{\pi^2}(2\pi) \right] = 0,$$

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$$a_{02} = \frac{1}{\pi} \left[-\frac{1}{4\pi}(1-1) + \frac{1}{\pi^2}(4\pi) \right] = \underline{\underline{\frac{1}{2\pi^2}}} = a_{02}$$

$$a_{03} = \frac{1}{\pi} \left[-\frac{1}{9\pi}(-1-1) + \frac{1}{27\pi^2}(-6\pi) \right] = 0.$$

$$a_{04} = \frac{1}{\pi} \left[-\frac{1}{16\pi}(0) + \frac{1}{64\pi^2}(8\pi) \right] = + \underline{\underline{\frac{1}{8\pi^2}}} = \underline{\underline{a_{04}}}$$

$$a_{05} = \frac{1}{\pi} \left[-\frac{1}{25\pi}(-1-1) + \frac{1}{125\pi^2}(-10\pi) \right] = 0$$

$$a_{06} = \frac{1}{\pi} \left[0 + \frac{1}{36 \times 6\pi^2}(12\pi) \right] = + \underline{\underline{\frac{1}{18\pi^2}}} = \underline{\underline{a_{06}}}$$

$$a_{40} = \frac{3}{8\pi^2}$$

$$a_{60} = \frac{3}{11\pi^2}$$

$$2 - \frac{x}{\pi} + 1$$

$$\begin{aligned}
a_{11} &= \frac{4}{\pi^2} \left[\int_0^\pi \int_0^{\pi-x} \left[\left(1 - \left(\frac{x}{\pi} \right)^2 \right) - \left(\frac{x}{\pi} \right) \left(1 + \left(\frac{x}{\pi} \right) \right) \right] \cos y \, dy \, \cos x \, dx \right. \\
&\quad \left. + \int_0^\pi \int_{\pi-x}^\pi \left(2 - \frac{x}{\pi} \right) \left(\frac{x}{\pi} - 1 + \frac{x}{\pi} \right) \cos y \, dy \, \cos x \, dx \right] \\
&= \frac{4}{\pi^2} \left[\int_0^\pi \int_0^{\pi-x} \left\{ \left(1 - \frac{x}{\pi} \right) - \left(\frac{x}{\pi} \right) \left(\frac{x}{\pi} \right) - \left(\frac{x}{\pi} \right)^2 \right\} \cos y \, dy \, \cos x \, dx \right. \\
&\quad \left. + \int_0^\pi \int_{\pi-x}^\pi \left\{ 2 \left(\frac{x}{\pi} - 1 \right) + \left(3 - \frac{x}{\pi} \right) \frac{x}{\pi} - \left(\frac{x}{\pi} \right)^2 \right\} \cos y \, dy \, \cos x \, dx \right] \\
&= \frac{4}{\pi^2} \left[\int_0^\pi \left\{ \left(1 - \frac{x}{\pi} \right) \sin x - \frac{x}{\pi} \frac{1}{\pi} \left(-\cos x - 1 + \sin x (\pi - x) \right) \right. \right. \\
&\quad \left. \left. - \frac{1}{\pi^2} \left(-2(\pi - x) \cos x + \frac{(\pi - x)^2}{2} \sin x \right) \right\} \cos x \, dx \right. \\
&\quad \left. + \int_0^\pi \left\{ -2 \left(\frac{x}{\pi} - 1 \right) \sin x + \left(3 - \frac{x}{\pi} \right) \frac{1}{\pi} \left(-1 + \cos x - (\pi - x) \sin x \right) \right. \right. \\
&\quad \left. \left. - \frac{1}{\pi^2} \left(-2\pi + 2(\pi - x) \cos x - \frac{(\pi - x)^2}{2} \sin x \right) \right\} \cos x \, dx \right] \\
&= \frac{4}{\pi^2} \int_0^\pi \left\{ 3 \left(1 - \frac{x}{\pi} \right) \sin x - \frac{x}{\pi} \frac{1}{\pi} (-2) + \frac{3}{\pi} \left(-1 + \cos x - (\pi - x) \sin x \right) \right\} \cos x \, dx \\
&= \frac{4}{\pi^2} \int_0^\pi \left\{ 3 \left(1 - \frac{x}{\pi} \right) \sin x + \frac{2}{\pi} \frac{x}{\pi} - \frac{3}{\pi} + \frac{3}{\pi} \cos x - 3 \left(1 - \frac{x}{\pi} \right) \sin x \right\} \cos x \, dx \\
&= \frac{4}{\pi^2} \int_0^\pi \frac{1}{\pi} \left\{ 2 \frac{x}{\pi} - 3 + 3 \cos x \right\} \cos x \, dx
\end{aligned}$$

$$a_{11} = \frac{4}{\pi^2} \left\{ \frac{3}{2} + \frac{2}{\pi^2} \int_0^\pi x \cos x dx \right\} = \underline{\underline{\frac{4}{\pi^2} \left\{ \frac{3}{2} - \frac{4}{\pi^2} \right\}}}$$

684

$$\begin{aligned} a_{21} &= \frac{4}{\pi^2} \int_0^\pi \frac{1}{\pi} \left\{ 2 \frac{x}{\pi} - 3 + 3 \cos x \right\} \cos x dx \\ &= \frac{8}{\pi^4} \int_0^\pi x \cos x dx = \frac{2}{\pi^2} \int_0^\pi t \cos t dt \\ &= \frac{2}{\pi^2} \left[0 \right] = 0 \end{aligned}$$

$$\begin{aligned} a_{31} &= \frac{4}{\pi^2} \int_0^\pi \frac{1}{\pi} \left\{ 2 \frac{x}{\pi} - 3 + 3 \cos x \right\} \cos 3x dx \\ &= \frac{8}{\pi^4} \int_0^\pi x \cos 3x dx = \frac{8}{9\pi^4} \int_0^\pi t \cos t dt \\ &= \underline{\underline{-\frac{16}{9\pi^2}}} \end{aligned}$$

$$\begin{aligned}
& \int_0^{\pi-x} \left\{ \left(1 - \frac{x}{\pi}\right) - \left(\frac{x}{\pi}\right) \frac{y}{\pi} - \left(\frac{y}{\pi}\right)^2 \right\} \cos 2y \, dy \\
& + \int_{\pi-x}^{\pi} \left\{ 2\left(\frac{x}{\pi} - 1\right) + \left(3 - \frac{x}{\pi}\right) \frac{y}{\pi} - \left(\frac{y}{\pi}\right)^2 \right\} \cos 2y \, dy \\
& = -\frac{1}{2} \frac{(1-\frac{x}{\pi})}{\pi^2} \sin 2x - \left(\frac{x}{\pi}\right) \frac{1}{4\pi} \left\{ (\cos 2x - 1) + (-2(\pi-x) \sin 2x) \right\} \\
& \quad - \frac{1}{8\pi^2} \left\{ 4(\pi-x) \cos 2x - \left(\frac{4(\pi-x)^2}{2} - 2 \right) \sin 2x \right\} \\
& + \left(\frac{x}{\pi} - 1 \right) \left\{ + \sin 2x \right\} + \left(3 - \frac{x}{\pi} \right) \frac{1}{4\pi} \left\{ 1 - \cos 2x + 2(\pi-x) \sin 2x \right\} \\
& \quad - \frac{1}{8\pi^2} \left\{ 4\pi - 4(\pi-x) \cos 2x + \left\{ 4(\pi-x)^2 - 2 \right\} \sin 2x \right\} \\
& = -\frac{3}{2} \frac{(1-\frac{x}{\pi})}{\pi^2} \sin 2x + \frac{3}{2} \frac{(1-\frac{x}{\pi})}{\pi^2} \sin 2x - \frac{1}{2\pi} + \frac{3}{4\pi} (1 - \cos 2x)
\end{aligned}$$

$$a_{12} = \frac{4}{\pi^2} \int_0^{\pi} -\frac{3}{4\pi} \cos 2x \cos x \, dx = 0$$

$$a_{22} = \frac{4}{\pi^2} \int_0^{\pi} -\frac{3}{4\pi} \cos^2 2x \, dx = -\frac{4}{\pi^2} \frac{3}{8} = -\frac{3}{2\pi^2}$$

$$a_{32} = 0.$$

$$\begin{aligned}
& \int_0^{\pi-x} \left\{ \left(1 - \frac{x}{\pi}\right) - \left(\frac{x}{\pi}\right) \frac{y}{\pi} - \left(\frac{y}{\pi}\right)^2 \right\} \cos 3y \, dy \\
& + \int_{\pi-x}^{\pi} \left\{ 2\left(\frac{x}{\pi} - 1\right) + \left(3 - \frac{x}{\pi}\right) \frac{y}{\pi} - \left(\frac{y}{\pi}\right)^2 \right\} \cos 3y \, dy \\
& = \frac{1}{3} \left(1 - \frac{x}{\pi}\right) \left\{ \sin 3x \right\} - \left(\frac{x}{\pi}\right) \frac{1}{9\pi} \left\{ -\cos 3x - 1 + 3(\pi-x) \sin 3x \right\} \\
& \quad - \frac{1}{27\pi^2} \left\{ -6(\pi-x) \cos 3x + \left\{ 9(\pi-x)^2 - 2 \right\} \sin 3x \right\} \\
& + \frac{2}{3} \left(\frac{x}{\pi} - 1\right) \left\{ -\sin 3x \right\} + \left(3 - \frac{x}{\pi}\right) \frac{1}{9\pi} \left\{ -1 + \cos 3x - 3(\pi-x) \sin 3x \right\} \\
& \quad - \frac{1}{27\pi^2} \left\{ -6\pi + 6(\pi-x) \cos 3x - \left\{ 9(\pi-x)^2 - 2 \right\} \sin 3x \right\} \\
& = \left(1 - \frac{x}{\pi}\right) \sin 3x + \frac{2}{9\pi} \frac{x}{\pi} - \frac{1}{3\pi} + \frac{1}{3\pi} \cos 3x - \left(1 - \frac{x}{\pi}\right) \sin 3x \\
& = -\frac{1}{3\pi} + \frac{1}{3\pi} \cos 3x + \frac{2}{9\pi} \frac{x}{\pi}
\end{aligned}$$

$$\begin{aligned}
a_{13} &= \frac{4}{\pi^2} \frac{2}{9\pi} \int_0^{\pi} \frac{x}{\pi} \cos x \, dx = \frac{8}{9\pi^4} \int_0^{\pi} x \cos x \, dx \\
&= -\frac{16}{9\pi^4}
\end{aligned}$$

$$a_{23} = \frac{4}{\pi^2} \frac{2}{9\pi} \int_0^{\pi} \frac{x}{\pi} \cos 2x \, dx = 0$$

$$a_{33} = \frac{4}{\pi^2} \int_0^{\pi} \left(\frac{1}{3\pi} \cos 3x + \frac{2}{9\pi} \frac{x}{\pi} \right) \cos 3x \, dx = \frac{4}{\pi^2} \left\{ \frac{1}{6} - \frac{2}{9\pi^2} \frac{2}{9} \right\}$$

$$a_{00} = \frac{f}{12}$$

$$a_{10} = 0$$

$$a_{20} = \frac{f}{\pi^2} \left(\frac{3}{2} \right)$$

$$a_{30} = 0$$

$$a_{40} = \frac{f}{\pi^2} \left(\frac{3}{8} \right)$$

$$a_{60} = \frac{f}{\pi^2} \left(\frac{3}{18} \right)$$

$$a_{01} = 0$$

$$a_{02} = \frac{f}{\pi^2} \left(\frac{1}{2} \right)$$

$$a_{03} = 0$$

$$a_{10} = \frac{f}{\pi^2} \left(\frac{1}{8} \right)$$

$$a_{16} = \frac{f}{\pi^2} \left(\frac{1}{18} \right)$$

$$a_{11} = \frac{f}{\pi^2} \left(6 - \frac{16}{\pi^2} \right), \quad a_{21} = 0,$$

$$a_{31} = \frac{f}{\pi^2} \left(-\frac{16}{9\pi^2} \right)$$

$$a_{12} = 0, \quad a_{12} = \frac{f}{\pi^2} \left(-\frac{3}{2} \right), \quad a_{32} = 0$$

$$a_{13} = \frac{f}{\pi^2} \left(-\frac{16}{9\pi^2} \right), \quad a_{23} = 0, \quad a_{33} = \frac{f}{\pi^2} \left(\frac{2}{3} - \frac{16}{81\pi^2} \right)$$

$$\frac{w}{R} = \frac{f}{4} \cos \frac{2\pi x}{R} + \frac{f}{16} \cos \frac{4\pi x}{R} + \frac{f}{36} \cos \frac{6\pi x}{R}$$

$$+ \frac{f}{12} \cos \frac{2\pi y}{R} + \frac{f}{48} \cos \frac{4\pi y}{R} + \frac{f}{108} \cos \frac{6\pi y}{R}$$

$$+ \left(1 - \frac{f}{3\pi^2} \right) \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} - \frac{f}{27\pi^2} \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} - \frac{f}{4} \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R}$$

$$- \frac{f}{27\pi^2} \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} + \left(\frac{1}{9} - \frac{f}{243\pi^2} \right) \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | π |
|----------------|---|------------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|------------------|------------------|------------------|--------------------|---------|
| $\cos \theta$ | 1 | 0.9659 | 0.8660 | 0.7071 | 0.5000 | 0.2588 | 0 | -0.2588 | -0.5000 | -0.7071 | -0.8660 | -0.9659 | -1.0000 |
| $\cos 2\theta$ | 1 | 0.8660 | 0.5000 | 0 | -0.5000 | -0.8660 | -1.0000 | -0.8660 | -0.5000 | 0 | +0.5000 | +0.8660 | 1 |
| $\cos 3\theta$ | 1 | 0.7071 | 0 | -0.7071 | -1.0000 | -0.7071 | 0 | +0.7071 | 1 | +0.7071 | 0 | -0.7071 | -1.0000 |
| $\cos 4\theta$ | 1 | 0.5000 | -0.5000 | -1.0000 | -0.5000 | +0.5000 | +1 | +0.5000 | -0.5000 | -1.0000 | -0.5000 | +0.5000 | 1 |
| $\cos 6\theta$ | 1 | 0 | -1 | 0 | 1 | 0 | -1 | 0 | 1 | 0 | -1 | 0 | 1 |

$$x=0$$

$$\frac{w}{R} = 0.3402778 + 0.69948911 \cos \frac{\pi x}{R} - 0.16666667 \cos \frac{2\pi x}{R} + 0.07775434 \cos \frac{3\pi x}{R} + 0.02083333 \cos \frac{4\pi x}{R} + 0.009259259 \cos \frac{6\pi x}{R}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | π |
|---------------|----------|------------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|------------------|------------------|------------------|--------------------|----------|
| $a_1 \cos$ | 0.69979 | 0.67593 | 0.6602 | 0.49682 | 0.34990 | 0.18111 | 0 | -0.18111 | -0.34990 | -0.49682 | -0.6602 | -0.67593 | -0.69979 |
| $b_1 \sin$ | -0.16667 | -0.16434 | -0.08334 | 0 | +0.08334 | +0.16434 | +0.16667 | +0.18434 | +0.08334 | 0 | -0.08334 | -0.16434 | -0.16667 |
| $a_3 \cos$ | +0.07775 | +0.05498 | 0 | -0.05498 | -0.07775 | -0.05498 | 0 | +0.05498 | +0.07775 | +0.05498 | 0 | -0.05498 | -0.07775 |
| $a_4 \cos$ | +0.02083 | +0.01042 | -0.01042 | -0.02083 | -0.01042 | +0.01042 | +0.02083 | +0.01042 | -0.01042 | -0.02083 | -0.01042 | +0.01042 | +0.02083 |
| $a_6 \cos$ | +0.00926 | 0 | -0.00926 | 0 | +0.00926 | 0 | -0.00926 | 0 | +0.00926 | 0 | -0.00926 | 0 | +0.00926 |
| $\frac{w}{R}$ | 0.7812 | 0.9373 | 0.8433 | 0.7593 | 0.6946 | 0.6212 | 0.5185 | 0.3689 | 0.1503 | -0.1204 | -0.3689 | -0.5246 | -0.5237 |

$$w_{avg} = 1.555088$$

$$\chi = \frac{\pi}{12} \frac{R}{m}$$

$$\frac{10}{R} = 0.2477500 + 0.6836958 \cos \frac{\pi R}{R} - 0.1321667 \cos \frac{2\pi R}{R} + 0.0622406 \cos \frac{3\pi R}{R} + 0.0208333 \cos \frac{4\pi R}{R} + 0.009259259 \cos \frac{5\pi R}{R}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | π |
|------------|----------|------------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|------------------|------------------|------------------|--------------------|----------|
| $a_1 \cos$ | 0.68370 | 0.66039 | 0.59208 | 0.48344 | 0.34185 | 0.17694 | 0 | -0.11674 | -0.34685 | -0.68344 | -0.59208 | -0.66039 | -0.68370 |
| $a_2 \cos$ | -0.13317 | -0.11532 | -0.06659 | 0 | 0.06659 | 0.11532 | 0.13317 | 0.15322 | 0.06659 | 0 | -0.06659 | -0.11532 | -0.13317 |
| $a_3 \cos$ | +0.04724 | +0.03338 | 0 | -0.03338 | -0.06421 | -0.03338 | 0 | +0.03338 | +0.06421 | +0.03338 | 0 | -0.03338 | -0.04724 |
| $a_4 \cos$ | | | | | | | | | | | | | |
| $a_5 \cos$ | | | | | | | | | | | | | |
| | 0.8756 | 0.8366 | 0.7536 | 0.6370 | 0.6078 | 0.5121 | 0.3925 | 0.2299 | 0.0165 | -0.2231 | -0.4306 | -0.5509 | -0.5862 |

$$\chi = \frac{\pi R}{6\pi}$$

$$\frac{u}{R} = 0.659722 + 0.6320156 \cos \frac{\pi}{4} - 0.041117 \cos \frac{3\pi}{4} - 0.13002109 \cos \frac{5\pi}{4} + 0.0208333 \cos \frac{7\pi}{4} + 0.0191595 \cos \frac{9\pi}{4}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | π |
|-----------|----------|------------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|------------------|------------------|------------------|--------------------|----------|
| u_{120} | 0.63202 | 0.61047 | 0.54233 | 0.44690 | 0.31601 | 0.16357 | 0 | -0.16357 | -0.31601 | -0.44690 | -0.54233 | -0.61047 | -0.63202 |
| u_{120} | -0.04117 | -0.03609 | -0.01044 | 0 | +0.01044 | +0.03609 | +0.04167 | +0.03609 | +0.02084 | 0 | -0.02084 | -0.03609 | -0.04167 |
| u_{130} | -0.03002 | -0.02123 | 0 | +0.02123 | +0.03102 | +0.02123 | 0 | -0.02123 | -0.03002 | -0.02123 | 0 | +0.02123 | +0.03102 |
| u_{140} | | | | | | | | | | | | | |
| u_{160} | | | | | | | | | | | | | |
| | 0.6564 | 0.6295 | 0.5724 | 0.5133 | 0.4317 | 0.2973 | 0.1192 | -0.0823 | -0.1604 | -0.4230 | -0.5219 | -0.5449 | -0.5476 |

$$\chi = \frac{\pi}{4} \frac{A}{R}$$

$$\frac{d\chi}{d\theta} = -0.0625000 + 0.5122467 \cos \frac{\pi}{6} \frac{A}{R} + 0.533333 \cos \frac{\pi}{3} \frac{A}{R} - 0.09459 \cos \frac{2\pi}{3} \frac{A}{R} + 0.00259259 \cos \frac{5\pi}{6} \frac{A}{R}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{5\pi}{6}$ | $\frac{3\pi}{4}$ | $\frac{7\pi}{8}$ | $\frac{11\pi}{8}$ | π |
|-------|-----------|------------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|------------------|-------------------|-----------|
| 0.000 | 0.53328 | 0.51296 | 0.48528 | 0.34991 | 0.26884 | 0.13905 | 0 | -0.13705 | -0.21164 | -0.34796 | -0.53328 |
| 0.008 | +0.008333 | +0.02216 | 0.04167 | 0 | -0.06111 | -0.08211 | -0.09333 | 0 | -0.04167 | -0.02216 | -0.008333 |
| 0.016 | -0.009744 | -0.00890 | 0 | +0.06190 | +0.07208 | +0.06190 | 0 | -0.06190 | -0.07208 | -0.06190 | +0.009744 |
| | 0.4908 | 0.4701 | 0.4408 | 0.3655 | 0.208 | 0.0837 | -0.1343 | -0.3322 | -0.4714 | -0.5321 | -0.4300 |

$$\chi = \frac{\pi}{3} \frac{A}{R}$$

$$\frac{d\chi}{d\theta} = -0.166722 + 0.394962 \cos \frac{\pi}{6} \frac{A}{R} + 0.001111 \cos \frac{2\pi}{3} \frac{A}{R} - 0.124860 \cos \frac{5\pi}{6} \frac{A}{R} + 0.010133 \cos \frac{7\pi}{8} \frac{A}{R} + 0.00959259 \cos \frac{9\pi}{8} \frac{A}{R}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | π |
|-------|----------|------------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|------------------|------------------|------------------|--------------------|----------|
| 0.000 | 0.39493 | 0.3814 | 0.3420 | 0.2796 | 0.1400 | 0.0221 | 0 | -0.1022 | -0.1924 | -0.2796 | -0.3420 | -0.3814 | -0.39493 |
| 0.008 | 0.1011 | 0.1204 | 0.147 | 0 | -0.0402 | -0.1044 | 0.2013 | 0.1101 | 0.1044 | 0 | -0.1044 | -0.1804 | -0.1011 |
| 0.016 | -0.11104 | -0.05221 | 0 | +0.00002 | +0.1000 | +0.1000 | 0 | -0.1000 | -0.1214 | -0.0880 | 0 | +0.0880 | +0.12274 |
| | 0.3430 | 0.350 | 0.360 | 0.380 | 0.390 | 0.395 | -0.395 | -0.400 | -0.350 | -0.250 | -0.160 | -0.080 | -0.0112 |

$$\chi = \frac{5H}{12} \frac{R}{m}$$

$$\frac{45}{R} = -0.165250 + 0.2401028 \cos \frac{mR}{R} + 0.2996333 \cos \frac{2mR}{R} - 0.0139774 \cos \frac{3mR}{R} + 0.0206333 \cos \frac{4mR}{R} + 0.029159259 \cos \frac{6mR}{R}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | π |
|--------------------|----------|------------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|------------------|------------------|------------------|--------------------|----------|
| $a_1 \cos 2\theta$ | 0.24010 | 0.24294 | 0.18195 | 0.14256 | 0.10505 | 0.05437 | 0 | -0.05437 | -0.10505 | -0.14256 | -0.18195 | -0.24294 | -0.24010 |
| $a_2 \cos 4\theta$ | 0.29963 | 0.25965 | 0.14992 | 0 | -0.14992 | -0.25965 | -0.29963 | -0.35965 | -0.14992 | 0 | 0.14992 | 0.25965 | 0.29963 |
| $a_3 \cos 6\theta$ | -0.01398 | -0.05938 | 0 | +0.05938 | +0.08398 | +0.05938 | 0 | -0.05938 | -0.08398 | -0.05938 | 0 | +0.05938 | +0.08398 |
| | 0.2708 | 0.2284 | 0.1269 | 0.0019 | -0.1473 | -0.3207 | -0.4735 | -0.5682 | -0.5154 | -0.4140 | -0.2440 | -0.0567 | +0.0186 |

$$\chi = \frac{\pi}{2} \frac{R}{m}$$

$$\frac{45}{R} = -0.2452378 + 0.23333 \cos \frac{mR}{R} + 0.0201333 \cos \frac{6mR}{R} + 0.029159259 \cos \frac{6mR}{R}$$

| | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | π |
|--------------------|---------|------------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|------------------|------------------|------------------|--------------------|----------|
| $a_2 \cos 2\theta$ | 0.13333 | 0.48666 | 0.16667 | 0 | -0.16667 | -0.48667 | -0.3333 | -0.16667 | -0.16667 | 0 | +0.16667 | +0.48667 | +0.33333 |
| | 0.1481 | 0.0838 | -0.0663 | -0.2361 | -0.431 | -0.49352 | -0.5370 | | | | | | |

$$\frac{w}{R} = f_0 + f_1 \left\{ 0.160763 \cos \frac{2\pi y}{R} + 0.040191 \cos \frac{4\pi y}{R} + 0.017863 \cos \frac{6\pi y}{R} \right. \\ \left. + 0.053588 \cos \frac{2\pi x}{R} + 0.013397 \cos \frac{4\pi x}{R} + 0.005954 \cos \frac{6\pi x}{R} \right. \\ \left. + 0.469305 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} - 0.160763 \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} - 0.019305 \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} \right. \\ \left. - 0.019305 \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} + 0.069305 \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R} \right\} + \frac{f_2}{2} \left\{ \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} \right\} \quad \underline{196}$$

$$\frac{w}{R} = f_0 + 3af_1 \cos \frac{2\pi y}{R} + 3bf_1 \cos \frac{4\pi y}{R} + 3cf_1 \cos \frac{6\pi y}{R} \\ + af_1 \cos \frac{2\pi x}{R} + bf_1 \cos \frac{4\pi x}{R} + cf_1 \cos \frac{6\pi x}{R} \\ + (\alpha f_1 + 0.5f_2) \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} - \beta f_1 \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} - \gamma f_1 \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} \\ - \gamma f_1 \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} + \delta f_1 \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R}$$

where

$$a = 0.053588, \quad b = 0.013397, \quad c = 0.005954 \\ \alpha = 0.469305, \quad \beta = 0.160763, \quad \gamma = 0.019305 \\ \delta = 0.069305$$

| | a | b | c | α | β | γ | δ |
|----------|------------|------------|------------|------------|------------|------------|------------|
| a | 0.0028717 | 0.00071792 | 0.00031906 | 0.0251491 | 0.00861497 | 0.0010345 | 0.0037139 |
| b | 0.00071792 | 0.00017948 | 0.00007977 | 0.00628728 | 0.00215374 | 0.00025863 | 0.00092848 |
| c | 0.00031906 | 0.00007977 | 0.00003545 | 0.00279424 | 0.00095718 | 0.00011494 | 0.00041264 |
| α | 0.0251491 | 0.00628728 | 0.00279424 | 0.2202472 | 0.07544688 | 0.00905993 | 0.03252518 |
| β | 0.00861497 | 0.00215374 | 0.00095718 | 0.07544688 | 0.2584424 | 0.00310353 | 0.01114168 |
| γ | 0.0010345 | 0.00025863 | 0.00011494 | 0.00905993 | 0.00310353 | 0.00037268 | 0.00133793 |
| δ | 0.0037139 | 0.00092848 | 0.00041264 | 0.03252518 | 0.01114168 | 0.00133793 | 0.00480318 |

$$\frac{\partial \omega}{\partial x} = -m \left\{ 6af_1 \sin \frac{2m}{R} + 12bf_1 \sin \frac{4m}{R} + 18cf_1 \sin \frac{6m}{R} + (\alpha f_1 + 0.5f_2) \sin \frac{2m}{R} \cos \frac{m}{R} - 2\beta f_1 \sin \frac{2m}{R} \cos \frac{3m}{R} \right. \\ \left. - 3\gamma f_1 \sin \frac{3m}{R} \cos \frac{m}{R} - \gamma f_1 \sin \frac{m}{R} \cos \frac{3m}{R} + 3\delta f_1 \sin \frac{3m}{R} \cos \frac{3m}{R} \right\}$$

$$\frac{\partial \omega}{\partial y} = -n \left\{ 2af_1 \sin \frac{2m}{R} + 4bf_1 \sin \frac{4m}{R} + 6cf_1 \sin \frac{6m}{R} + (\alpha f_1 + 0.5f_2) \cos \frac{2m}{R} \sin \frac{m}{R} - 2\beta f_1 \cos \frac{2m}{R} \sin \frac{3m}{R} \right. \\ \left. - \gamma f_1 \cos \frac{3m}{R} \sin \frac{m}{R} - 3\gamma f_1 \cos \frac{m}{R} \sin \frac{3m}{R} + 3\delta f_1 \cos \frac{3m}{R} \sin \frac{3m}{R} \right\}$$

$$\frac{\partial^2 \omega}{\partial x^2} = -\frac{m^2}{R} \left\{ 12af_1 \cos \frac{2m}{R} + 48bf_1 \cos \frac{4m}{R} + 108cf_1 \cos \frac{6m}{R} + (\alpha f_1 + 0.5f_2) \cos \frac{2m}{R} \cos \frac{m}{R} - 4\beta f_1 \cos \frac{2m}{R} \cos \frac{3m}{R} \right. \\ \left. - 9\gamma f_1 \cos \frac{3m}{R} \cos \frac{m}{R} - \gamma f_1 \cos \frac{m}{R} \cos \frac{3m}{R} + 9\delta f_1 \cos \frac{3m}{R} \cos \frac{3m}{R} \right\}$$

$$\frac{\partial^2 \omega}{\partial y^2} = -\frac{m^2}{R} \left\{ 4af_1 \cos \frac{2m}{R} + 16bf_1 \cos \frac{4m}{R} + 36cf_1 \cos \frac{6m}{R} + (\alpha f_1 + 0.5f_2) \cos \frac{m}{R} \cos \frac{m}{R} - 4\beta f_1 \cos \frac{2m}{R} \cos \frac{3m}{R} \right. \\ \left. - \gamma f_1 \cos \frac{3m}{R} \cos \frac{m}{R} - 9\gamma f_1 \cos \frac{m}{R} \cos \frac{3m}{R} + 9\delta f_1 \cos \frac{3m}{R} \cos \frac{3m}{R} \right\}$$

$$\frac{\partial^2 \omega}{\partial x \partial y} = \frac{mn}{R} \left\{ (\alpha f_1 + 0.5f_2) \sin \frac{m}{R} \cos \frac{m}{R} - 4\beta f_1 \sin \frac{2m}{R} \sin \frac{3m}{R} - 3\gamma f_1 \sin \frac{3m}{R} \sin \frac{m}{R} \right. \\ \left. - 3\gamma f_1 \sin \frac{m}{R} \sin \frac{3m}{R} + 9\delta f_1 \sin \frac{3m}{R} \sin \frac{3m}{R} \right\}$$

$$\begin{aligned}
I = & \frac{m^4}{R^2} \int \frac{1}{4} (\alpha f_1 + 0.5 f_2)^2 \left(1 - \cos \frac{2\pi y}{R} \right) + 4 \beta^2 f_1^2 \left(1 - \cos \frac{4\pi y}{R} \right) / \left(1 - \cos \frac{4\pi y}{R} \right) \\
& + \frac{9}{4} \gamma f_1^2 \left(1 - \cos \frac{6\pi y}{R} \right) / \left(1 - \cos \frac{2\pi y}{R} \right) + \frac{9}{4} \gamma f_1^2 \left(1 - \cos \frac{2\pi y}{R} \right) / \left(1 - \cos \frac{6\pi y}{R} \right) + \frac{5}{4} \delta^2 f_1^2 \left(1 - \cos \frac{6\pi y}{R} \right) / \left(1 - \cos \frac{6\pi y}{R} \right) \\
& - 2 \beta f_1 (\alpha f_1 + 0.5 f_2) \left(\cos \frac{\pi y}{R} - \cos \frac{3\pi y}{R} \right) / \left(\cos \frac{\pi y}{R} - \cos \frac{3\pi y}{R} \right) - \frac{3}{2} \gamma f_1 (\alpha f_1 + 0.5 f_2) / \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) / \left(1 - \cos \frac{2\pi y}{R} \right) \\
& - \frac{3}{2} \gamma f_1 (\alpha f_1 + 0.5 f_2) \left(1 - \cos \frac{2\pi y}{R} \right) / \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) + \frac{9}{2} \delta f_1 (\alpha f_1 + 0.5 f_2) \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) / \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) \\
& + 6 \beta \gamma f_1^2 \left(\cos \frac{\pi y}{R} - \cos \frac{5\pi y}{R} \right) / \left(\cos \frac{\pi y}{R} - \cos \frac{3\pi y}{R} \right) + 6 \beta \gamma f_1^2 \left(\cos \frac{\pi y}{R} - \cos \frac{3\pi y}{R} \right) / \left(\cos \frac{\pi y}{R} - \cos \frac{5\pi y}{R} \right) \\
& - 18 \beta \delta f_1^2 \left(\cos \frac{\pi y}{R} - \cos \frac{5\pi y}{R} \right) / \left(\cos \frac{\pi y}{R} - \cos \frac{5\pi y}{R} \right) + \frac{9}{2} \gamma f_1^2 \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) / \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) \\
& - \frac{27}{2} \gamma \delta f_1^2 \left(1 - \cos \frac{6\pi y}{R} \right) / \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) - \frac{27}{2} \gamma \delta f_1^2 \left(\cos \frac{2\pi y}{R} - \cos \frac{4\pi y}{R} \right) / \left(1 - \cos \frac{6\pi y}{R} \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} (\alpha f_1 + 0.5 f_2)^2 \left(1 + c_{\omega} \frac{2mY}{R} \right) \left(1 + c_{\omega} \frac{4mY}{R} \right) - 4\beta^2 f_1^2 \left(1 + c_{\omega} \frac{4mY}{R} \right) - \frac{9}{4} \beta^2 f_1^2 \left(1 + c_{\omega} \frac{6mY}{R} \right) \left(1 + c_{\omega} \frac{2mY}{R} \right) \\
& - \frac{9}{4} \beta^2 f_1^2 \left(1 + c_{\omega} \frac{2mY}{R} \right) \left(1 + c_{\omega} \frac{6mY}{R} \right) - \frac{81}{4} \beta^2 f_1^2 \left(1 + c_{\omega} \frac{6mY}{R} \right) \left(1 + c_{\omega} \frac{6mY}{R} \right) \\
& + 2\beta f_1 (\alpha f_1 + 0.5 f_2) \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{3mY}{R} \right) \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{3mY}{R} \right) + \frac{5}{2} \beta f_1 (\alpha f_1 + 0.5 f_2) \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{4mY}{R} \right) \left(1 + c_{\omega} \frac{2mY}{R} \right) \\
& + \frac{5}{2} \beta f_1 (\alpha f_1 + 0.5 f_2) \left(1 + c_{\omega} \frac{2mY}{R} \right) \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{6mY}{R} \right) - \frac{9}{2} \beta f_1 (\alpha f_1 + 0.5 f_2) \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{4mY}{R} \right) \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{4mY}{R} \right) \\
& - 10\beta \gamma f_1^2 \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{5mY}{R} \right) \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{3mY}{R} \right) - 10\beta \gamma f_1^2 \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{3mY}{R} \right) \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{5mY}{R} \right) \\
& + 18\beta \delta f_1^2 \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{5mY}{R} \right) \left(c_{\omega} \frac{mY}{R} + c_{\omega} \frac{5mY}{R} \right) - \frac{41}{2} \beta^2 f_1^2 \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{4mY}{R} \right) \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{4mY}{R} \right) \\
& + \frac{45}{2} \beta \delta f_1^2 \left(1 + c_{\omega} \frac{6mY}{R} \right) \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{4mY}{R} \right) + \frac{45}{2} \beta \delta f_1^2 \left(c_{\omega} \frac{2mY}{R} + c_{\omega} \frac{4mY}{R} \right) \left(1 + c_{\omega} \frac{6mY}{R} \right)
\end{aligned}$$

$$\begin{aligned}
I = & -\frac{m^4}{R^2} \left\{ \frac{1}{2} (\alpha f_1' + 0.5 f_2')^2 \left(\cos \frac{2\pi x}{R} + \cos \frac{4\pi x}{R} \right) + 8 \beta f_1'^2 \left(\cos \frac{4\pi x}{R} + \cos \frac{6\pi x}{R} \right) + \frac{9}{2} f_1'^2 \left(\cos \frac{6\pi x}{R} + \cos \frac{8\pi x}{R} \right) \right. \\
& + \frac{9}{2} f_1'^2 f_2'^2 \left(\cos \frac{2\pi x}{R} + \cos \frac{6\pi x}{R} \right) + \frac{41}{2} f_1'^2 f_2'^2 \left(\cos \frac{6\pi x}{R} + \cos \frac{10\pi x}{R} \right) - 4 \beta f_1' (\alpha f_1' + 0.5 f_2') \left(\cos \frac{3\pi x}{R} + \cos \frac{9\pi x}{R} \right) \\
& - f_1' (\alpha f_1' + 0.5 f_2') \left(\cos \frac{2\pi x}{R} + 4 \cos \frac{4\pi x}{R} + 4 \cos \frac{6\pi x}{R} + \cos \frac{8\pi x}{R} + \cos \frac{10\pi x}{R} \right) \\
& - f_1' (\alpha f_1' + 0.5 f_2') \left(\cos \frac{2\pi x}{R} + 4 \cos \frac{4\pi x}{R} + 4 \cos \frac{6\pi x}{R} + \cos \frac{8\pi x}{R} + \cos \frac{10\pi x}{R} \right) \\
& + 9 f_1' (\alpha f_1' + 0.5 f_2') \left(\cos \frac{4\pi x}{R} + \cos \frac{6\pi x}{R} + \cos \frac{8\pi x}{R} \right) + 4 \beta f_1'^2 \left(\cos \frac{4\pi x}{R} + 4 \cos \frac{6\pi x}{R} + 4 \cos \frac{8\pi x}{R} \right) \\
& + 4 \cos \frac{2\pi x}{R} + \cos \frac{10\pi x}{R} + 4 \cos \frac{12\pi x}{R} + 4 \beta f_1' f_2' \left(\cos \frac{2\pi x}{R} + \cos \frac{6\pi x}{R} + \cos \frac{10\pi x}{R} \right) + 4 \cos \frac{3\pi x}{R} + 4 \cos \frac{9\pi x}{R} \\
& + \cos \frac{3\pi x}{R} \cos \frac{5\pi x}{R} + 36 \beta f_1'^2 \left(\cos \frac{5\pi x}{R} + \cos \frac{7\pi x}{R} + \cos \frac{9\pi x}{R} \right) + f_1'^2 \left(16 \cos \frac{2\pi x}{R} + 25 \cos \frac{4\pi x}{R} + 25 \cos \frac{6\pi x}{R} \right. \\
& + 25 \cos \frac{8\pi x}{R} + 16 \cos \frac{10\pi x}{R} + 16 \cos \frac{12\pi x}{R} + 4 \cos \frac{14\pi x}{R} + 4 \cos \frac{16\pi x}{R} + 4 \cos \frac{18\pi x}{R} + \cos \frac{20\pi x}{R} \left. \right) \\
& - 9 f_1' f_2'^2 \left(\cos \frac{2\pi x}{R} + 4 \cos \frac{4\pi x}{R} + \cos \frac{6\pi x}{R} + 4 \cos \frac{8\pi x}{R} + \cos \frac{10\pi x}{R} + \cos \frac{12\pi x}{R} \right) \left. \right\}
\end{aligned}$$

$$\begin{aligned}
II = & -\frac{\pi^2}{R^2} \left\{ 48 a^2 f_1^2 \cos \frac{2\pi y}{R} \cos \frac{2\pi x}{R} + 192 a b f_1^2 \cos \frac{4\pi y}{R} \cos \frac{2\pi x}{R} + 432 a c f_1^2 \cos \frac{6\pi y}{R} \cos \frac{2\pi x}{R} \right. \\
& + 2 a f_1^2 (\alpha f_1 + 0.5 f_2) \left(\cos \frac{\pi y}{R} \cos \frac{\pi x}{R} + \cos \frac{3\pi y}{R} \cos \frac{3\pi x}{R} \right) - 8 a \beta f_1^2 \left(\cos \frac{2\pi y}{R} + \cos \frac{2\pi x}{R} \cos \frac{4\pi y}{R} \right) \\
& - 18 a \gamma f_1^2 \left(\cos \frac{3\pi y}{R} \cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R} \right) - 3 a \gamma f_1^2 \left(\cos \frac{\pi y}{R} \cos \frac{\pi x}{R} + \cos \frac{2\pi y}{R} \cos \frac{5\pi x}{R} \right) \\
& + 18 a \delta f_1^2 \left(\cos \frac{3\pi y}{R} \cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} \cos \frac{5\pi y}{R} \right) + 192 a b f_1^2 \cos \frac{2\pi y}{R} \cos \frac{4\pi x}{R} + 468 b^2 f_1^2 \cos \frac{4\pi y}{R} \cos \frac{4\pi x}{R} \\
& + 1728 b c f_1^2 \cos \frac{6\pi y}{R} \cos \frac{4\pi x}{R} + 8 b f_1^2 (\alpha f_1 + 0.5 f_2) \left(\cos \frac{\pi y}{R} \cos \frac{3\pi x}{R} + \cos \frac{\pi x}{R} \cos \frac{5\pi y}{R} \right) \\
& - 32 b \beta f_1^2 \left(\cos \frac{2\pi y}{R} \cos \frac{2\pi x}{R} + \cos \frac{2\pi x}{R} \cos \frac{6\pi y}{R} \right) - 32 b \gamma f_1^2 \left(\cos \frac{3\pi y}{R} \cos \frac{3\pi x}{R} + \cos \frac{3\pi x}{R} \cos \frac{5\pi y}{R} \right) \\
& - 8 b \gamma f_1^2 \left(\cos \frac{\pi y}{R} \cos \frac{\pi x}{R} + \cos \frac{\pi x}{R} \cos \frac{7\pi y}{R} \right) + 72 b \delta f_1^2 \left(\cos \frac{3\pi y}{R} \cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R} \cos \frac{7\pi y}{R} \right) \\
& + 342 a c f_1^2 \cos \frac{2\pi y}{R} \cos \frac{6\pi x}{R} + 1728 b c f_1^2 \cos \frac{4\pi y}{R} \cos \frac{6\pi x}{R} + 3888 c^2 f_1^2 \cos \frac{6\pi y}{R} \cos \frac{6\pi x}{R} \\
& + 18 c f_1^2 (\alpha f_1 + 0.5 f_2) \left(\cos \frac{\pi y}{R} \cos \frac{5\pi x}{R} + \cos \frac{\pi x}{R} \cos \frac{7\pi y}{R} \right) - 72 c \beta f_1^2 \left(\cos \frac{2\pi y}{R} \cos \frac{4\pi x}{R} + \cos \frac{2\pi x}{R} \cos \frac{8\pi y}{R} \right) \\
& - 162 c \gamma f_1^2 \left(\cos \frac{3\pi y}{R} \cos \frac{5\pi x}{R} + \cos \frac{3\pi x}{R} \cos \frac{7\pi y}{R} \right) - 18 c \gamma f_1^2 \left(\cos \frac{\pi y}{R} \cos \frac{3\pi x}{R} + \cos \frac{\pi x}{R} \cos \frac{9\pi y}{R} \right) \\
& + 162 c \delta f_1^2 \left(\cos \frac{3\pi y}{R} \cos \frac{3\pi x}{R} + \cos \frac{3\pi x}{R} \cos \frac{9\pi y}{R} \right)
\end{aligned}$$

Card 111

Cont'd !!!

$$\begin{aligned}
 & + 6af_1'(x f_1' + 0.5 f_2') \left(\cos \frac{\pi y}{R} \cos \frac{\pi y}{R} + \cos \frac{3\pi y}{R} \cos \frac{\pi y}{R} \right) + 24bf_1'(x f_1' + 0.5 f_2') \left(\cos \frac{2\pi y}{R} \cos \frac{\pi y}{R} + \cos \frac{5\pi y}{R} \cos \frac{\pi y}{R} \right) \\
 & + 54cf_1'(x f_1' + 0.5 f_2') \left(\cos \frac{5\pi y}{R} \cos \frac{\pi y}{R} + \cos \frac{7\pi y}{R} \cos \frac{\pi y}{R} \right) - 24af_1'^2 \left(\cos \frac{2\pi y}{R} + \cos \frac{4\pi y}{R} \cos \frac{2\pi y}{R} \right) \\
 & - 96bf_1'^2 \left(\cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} + \cos \frac{6\pi y}{R} \cos \frac{2\pi y}{R} \right) - 216cf_1'^2 \left(\cos \frac{4\pi y}{R} \cos \frac{2\pi y}{R} + \cos \frac{8\pi y}{R} \cos \frac{2\pi y}{R} \right) \\
 & - 6af_1'^2 \left(\cos \frac{\pi y}{R} + \cos \frac{5\pi y}{R} \cos \frac{\pi y}{R} \right) - 24bf_1'^2 \left(\cos \frac{2\pi y}{R} \cos \frac{\pi y}{R} + \cos \frac{7\pi y}{R} \cos \frac{\pi y}{R} \right) \\
 & - 54cf_1'^2 \left(\cos \frac{3\pi y}{R} \cos \frac{\pi y}{R} + \cos \frac{9\pi y}{R} \cos \frac{\pi y}{R} \right) - 54af_1'^2 \left(\cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} + \cos \frac{3\pi y}{R} \cos \frac{3\pi y}{R} \right) \\
 & - 216bf_1'^2 \left(\cos \frac{3\pi y}{R} \cos \frac{3\pi y}{R} + \cos \frac{5\pi y}{R} \cos \frac{3\pi y}{R} \right) - 486cf_1'^2 \left(\cos \frac{5\pi y}{R} \cos \frac{3\pi y}{R} + \cos \frac{7\pi y}{R} \cos \frac{3\pi y}{R} \right) \\
 & + 54a\delta f_1'^2 \left(\cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} + \cos \frac{5\pi y}{R} \cos \frac{3\pi y}{R} \right) + 216b\delta f_1'^2 \left(\cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} + \cos \frac{7\pi y}{R} \cos \frac{3\pi y}{R} \right) \\
 & + 486c\delta f_1'^2 \left(\cos \frac{3\pi y}{R} \cos \frac{3\pi y}{R} + \cos \frac{9\pi y}{R} \cos \frac{3\pi y}{R} \right) \}
 \end{aligned}$$

$$\text{III} = + \frac{m^2}{R^2} \left\{ 12af_1 \sqrt{\cos \frac{2m\pi}{R}} + 48bf_1 \cos \frac{4m\pi}{R} + 108cf_1 \cos \frac{6m\pi}{R} + (af_1 + 0.5f_2) \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} \right. \\ \left. - 4\beta f_1 \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} - 9\beta f_1 \cos \frac{2m\pi}{R} \cos \frac{m\pi}{R} - \beta f_1 \cos \frac{2m\pi}{R} \cos \frac{3m\pi}{R} + 9\delta f_1 \cos \frac{3m\pi}{R} \cos \frac{3m\pi}{R} \right\}$$

$$1 \left[\cos \frac{2m\pi}{R} \right] m^2 \left\{ \frac{1}{2} (af_1 + 0.5f_2)^2 + \frac{9}{2} \beta^2 f_1^2 - \beta f_1 (af_1 + 0.5f_2) - 9\delta f_1^2 - 8a\beta f_1^2 \right\} - 12af_1$$

$$2 \left[\cos \frac{4m\pi}{R} \right] m^2 \left\{ 8\beta^2 f_1^2 - 4\beta f_1 (af_1 + 0.5f_2) - 36\gamma\delta f_1^2 \right\} - 48bf_1$$

$$3 \left[\cos \frac{6m\pi}{R} \right] m^2 \left\{ \frac{9}{2} \beta^2 f_1^2 + \frac{\beta}{2} \delta^2 f_1^2 \right\} - 108cf_1$$

$$4 \left[\cos \frac{2m\pi}{R} \right] m^2 \left\{ \frac{1}{2} (af_1 + 0.5f_2)^2 + \frac{9}{2} \beta^2 f_1^2 - \beta f_1 (af_1 + 0.5f_2) - 9\delta f_1^2 - 24a\beta f_1^2 \right\}$$

$$5 \left[\cos \frac{4m\pi}{R} \right] m^2 \left\{ 8\beta^2 f_1^2 - 4\beta f_1 (af_1 + 0.5f_2) - 36\gamma\delta f_1^2 \right\}$$

$$6 \left[\cos \frac{6m\pi}{R} \right] m^2 \left\{ \frac{9}{2} \beta^2 f_1^2 + \frac{\beta}{2} \delta^2 f_1^2 \right\}$$

$$7 \left[\cos \frac{m\pi}{R} \cos \frac{m\pi}{R} \right] m^2 \left\{ 4\beta\delta f_1^2 + 4\beta\gamma f_1^2 + 2af_1 (af_1 + 0.5f_2) - 2a\gamma f_1^2 - 86\gamma f_1^2 + 6a\delta (af_1 + 0.5f_2) \right. \\ \left. - 6a\gamma f_1^2 - 246\gamma f_1^2 \right\} - (af_1 + 0.5f_2)$$

$$I \left[\cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} \right] \cdot m^2 \left\{ -4\gamma f_1 (\alpha f_1 + 0.5 f_2) - 4\gamma f_1 (\alpha f_1 + 0.5 f_2) + 16\gamma^2 f_1^2 + 48\alpha^2 f_1^2 - 326\beta f_1^2 - 966\beta f_1^2 \right\} + 4\beta f_1$$

$$II \left[\cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} \right] \cdot m^2 \left\{ -4\beta f_1 (\alpha f_1 + 0.5 f_2) + 16\beta \gamma f_1^2 - 18\alpha \gamma f_1^2 + 18\alpha \delta f_1^2 + 326\delta f_1^2 + 6\alpha f_1 (\alpha f_1 + 0.5 f_2) + 246\alpha f_1 (\alpha f_1 + 0.5 f_2) - 54c\gamma f_1^2 \right\} + 9\gamma f_1$$

$$III \left[\cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} \right] \cdot m^2 \left\{ -4\beta f_1 (\alpha f_1 + 0.5 f_2) + 16\beta \gamma f_1^2 + 2\alpha f_1 (\alpha f_1 + 0.5 f_2) + 86\alpha f_1 (\alpha f_1 + 0.5 f_2) - 18c\gamma f_1^2 - 54\alpha \gamma f_1^2 + 54\alpha \delta f_1^2 + 216\delta f_1^2 \right\} + \gamma f_1$$

$$IV \left[\cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R} \right] \cdot m^2 \left\{ -18\alpha \gamma f_1^2 - 326\gamma f_1^2 + 162c\delta f_1^2 - 54\alpha \gamma f_1^2 - 216\beta \gamma f_1^2 + 486c\delta f_1^2 \right\} + 9\delta f_1$$

$$I \left[\cos \frac{2\pi\nu}{R} \cos \frac{5\pi\nu}{R} \right] : m^2 \left\{ -36\beta\delta f_1^2 - 24\gamma f_1^2 + 86c_1^2 (\alpha f_1 + 0.5f_2) + 18c_1^2 (\alpha f_1 + 0.5f_2) + 16\beta\gamma f_1^2 \right\}$$

$$II \left[\cos \frac{5\pi\nu}{R} \cos \frac{7\pi\nu}{R} \right] : m^2 \left\{ 16\beta\gamma f_1^2 - 36\beta\delta f_1^2 + 24\gamma f_1^2 (\alpha f_1 + 0.5f_2) + 54c_1^2 (\alpha f_1 + 0.5f_2) - 64\gamma f_1^2 \right\}$$

$$III \left[\cos \frac{2\pi\nu}{R} \cos \frac{4\pi\nu}{R} \right] : m^2 \left\{ -\gamma f_1^2 (\alpha f_1 + 0.5f_2) + 9\delta f_1^2 (\alpha f_1 + 0.5f_2) + 25\gamma^2 f_1^2 - 84\beta\gamma f_1^2 + 192\alpha\beta f_1^2 - 72c\beta\gamma f_1^2 \right\}$$

$$IV \left[\cos \frac{4\pi\nu}{R} \cos \frac{2\pi\nu}{R} \right] : m^2 \left\{ -\gamma f_1^2 (\alpha f_1 + 0.5f_2) + 9\delta f_1^2 (\alpha f_1 + 0.5f_2) + 192\alpha\beta f_1^2 - 244\beta\gamma f_1^2 - 25\gamma^2 f_1^2 \right\}$$

$$V \left[\cos \frac{7\pi\nu}{R} \cos \frac{7\pi\nu}{R} \right] : m^2 \left\{ -86\gamma f_1^2 + 18c_1^2 (\alpha f_1 + 0.5f_2) \right\}$$

$$VI \left[\cos \frac{7\pi\nu}{R} \cos \frac{7\pi\nu}{R} \right] : m^2 \left\{ 54c_1^2 (\alpha f_1 + 0.5f_2) - 246\gamma f_1^2 \right\}$$

$$VII \left[\cos \frac{2\pi\nu}{R} \cos \frac{6\pi\nu}{R} \right] : m^2 \left\{ -36\gamma\delta f_1^2 - 326\beta\gamma f_1^2 + 342\alpha c f_1^2 \right\}$$

$$VIII \left[\cos \frac{6\pi\nu}{R} \cos \frac{2\pi\nu}{R} \right] : m^2 \left\{ -36\gamma\delta f_1^2 + 432\alpha c f_1^2 - 966\beta\gamma f_1^2 \right\}$$

$$\begin{aligned}
20 & \left[\cos \frac{3\pi x}{R} \cos \frac{5\pi y}{R} \right] : m^2 \left\{ 4\beta\delta f_1'^2 + 18\alpha\delta f_1'^2 - 12\beta\gamma f_1'^2 - 162\alpha\gamma f_1'^2 \right\} \\
21 & \left[\cos \frac{5\pi x}{R} \cos \frac{3\pi y}{R} \right] : m^2 \left\{ 4\beta\delta f_1'^2 - 216\beta\gamma f_1'^2 + 54\alpha\delta f_1'^2 - 486\alpha\gamma f_1'^2 \right\} \\
22 & \left[\cos \frac{4\pi x}{R} \cos \frac{4\pi y}{R} \right] : m^2 \left\{ 16\gamma f_1'^2 + 72\delta f_1'^2 \right\} \\
23 & \left[\cos \frac{\pi x}{R} \cos \frac{9\pi y}{R} \right] : m^2 \left\{ -18\alpha\gamma f_1'^2 \right\} \\
24 & \left[\cos \frac{9\pi x}{R} \cos \frac{\pi y}{R} \right] : m^2 \left\{ -54\alpha\gamma f_1'^2 \right\} \\
25 & \left[\cos \frac{6\pi x}{R} \cos \frac{6\pi y}{R} \right] : m^2 \left\{ -9\gamma\delta f_1'^2 + 1728\beta\alpha f_1'^2 \right\} \\
26 & \left[\cos \frac{6\pi x}{R} \cos \frac{6\pi y}{R} \right] : m^2 \left\{ -9\gamma\delta f_1'^2 + 1728\beta\alpha f_1'^2 \right\} \\
27 & \left[\cos \frac{3\pi x}{R} \cos \frac{7\pi y}{R} \right] : m^2 \left\{ 72\beta\delta f_1'^2 - 162\alpha\gamma f_1'^2 \right\} \\
28 & \left[\cos \frac{7\pi x}{R} \cos \frac{3\pi y}{R} \right] : m^2 \left\{ -486\alpha\gamma f_1'^2 + 216\beta\delta f_1'^2 \right\} \\
29 & \left[\cos \frac{2\pi x}{R} \cos \frac{8\pi y}{R} \right] : m^2 \left\{ -72\alpha\beta f_1'^2 \right\} \\
30 & \left[\cos \frac{8\pi x}{R} \cos \frac{2\pi y}{R} \right] : m^2 \left\{ -216\alpha\beta f_1'^2 \right\}
\end{aligned}$$

$$\begin{aligned}
31 & \left[\cos \frac{6\pi x}{R} \cos \frac{6\pi y}{R} \right] : m^2 \left\{ 3888\alpha^2 f_1'^2 \right\} \\
32 & \left[\cos \frac{3\pi x}{R} \cos \frac{9\pi y}{R} \right] : m^2 \left\{ 162\alpha\delta f_1'^2 \right\} \\
33 & \left[\cos \frac{9\pi x}{R} \cos \frac{3\pi y}{R} \right] : m^2 \left\{ 486\alpha\delta f_1'^2 \right\}
\end{aligned}$$

206

$$\begin{aligned}
1 &= m^2 \left\{ f_1^2 \left(\frac{1}{2} \alpha^2 + \frac{9}{2} f^2 - \alpha\gamma - 9\gamma\delta - 8\alpha\beta \right) + f_1 f_2 \left(\frac{1}{2} \alpha - 0.5\gamma \right) + \frac{1}{8} f_1^2 \right\} - 12\alpha f_1 \\
&= m^2 \left\{ 0.0242796 f_1^2 + 0.225 f_1 f_2 + 0.125000 f_2^2 \right\} - 0.643056 f_1 \quad \left(\frac{1}{8} \right) \\
2 &= m^2 \left\{ f_1^2 (8\beta^2 - 4\alpha\gamma - 36\gamma\delta) - f_1 f_2 (2\gamma) \right\} - 48\delta f_1 \\
&= m^2 \left\{ 0.122353 f_1^2 - 0.038610 f_1 f_2 \right\} - 0.643056 f_1 \quad \left(\frac{1}{16} \right) \\
3 &= m^2 \left\{ 0.196206 f_1^2 \right\} - 0.643056 f_1 \quad \left(\frac{1}{48} \right) \\
4 &= m^2 \left\{ \left(\frac{1}{2} \alpha^2 + \frac{9}{2} f^2 - \alpha\gamma - 9\gamma\delta - 24\alpha\beta \right) f_1^2 + \left(\frac{1}{2} \alpha - 0.5\gamma \right) f_1 f_2 + \frac{1}{8} f_2^2 \right\} \\
&= m^2 \left\{ -0.1160599 f_1^2 + 0.225 f_1 f_2 + 0.125000 f_2^2 \right\} \quad \left(\frac{1}{8} \right) \\
5 &= m^2 \left\{ 0.122353 f_1^2 - 0.038610 f_1 f_2 \right\} \quad \left(\frac{1}{16} \right) \\
6 &= m^2 \left\{ 0.196206 f_1^2 \right\} \quad \left(\frac{1}{48} \right) \\
7 &= m^2 \left\{ f_1^2 (8\beta\gamma + 6\alpha\delta - 8\alpha\gamma - 32\gamma\delta) + 3\alpha f_1 f_2 \right\} - (\alpha f_1 + 0.5 f_2) \\
&= m^2 \left\{ 0.159171 f_1^2 + 0.160264 f_1 f_2 \right\} - (0.469305 f_1 + 0.5000 f_2) \quad \left(\frac{1}{4} \right)
\end{aligned}$$

$$r = m^2 \left\{ (-8\alpha\gamma + 16\gamma^2 + 48\alpha^2 - 1086\beta) f_1^2 - 4\gamma f_1 f_2 \right\} + 48 f_1^2$$

$$= - \left[m^2 \left\{ 0.161249 f_1^2 + 0.077220 f_1 f_2 \right\} - 0.643052 f_1^2 \right] \quad \left(\frac{1}{11} \right)$$

$$g = m^2 \left\{ (-4\alpha\beta + 16\beta\gamma - 18\alpha\gamma + 18\alpha\gamma + 726\delta + 64\alpha + 246\alpha - 540\gamma) f_1^2 + (-2\beta + 3\alpha + 126) f_1 f_2 \right\} + 98 f_1^2$$

$$= m^2 \left\{ 0.158531 f_1^2 + 0.000002 f_1 f_2 \right\} + 0.173765 f_1^2 \quad \left(\frac{1}{100} \right)$$

$$11) = m^2 \left\{ (-4\alpha\beta + 16\beta\gamma - 54\alpha\gamma + 540\delta + 2466\delta + 20\alpha + 86\alpha - 180\gamma) f_1^2 + (-2\beta + \alpha + 46) f_1 f_2 \right\} + 8 f_1^2$$

$$= m^2 \left\{ 0.191636 f_1^2 - 0.21435 f_1 f_2 \right\} + 0.019305 f_1^2 \quad \left(\frac{1}{100} \right)$$

$$11 = m^2 \left\{ (-22\alpha\gamma - 2486\gamma + 6480\delta) f_1^2 + 98 f_1^2 \right\}$$

$$= m^2 \left\{ 0.118421 f_1^2 \right\} + 0.623745 f_1^2 \quad \left(\frac{1}{324} \right)$$

708

$$12 = m^2 \left\{ f_1^2 (-36\beta\delta - 2\alpha\gamma + 86\alpha + 18c\alpha + 16\beta\gamma) + f_1 f_2 (46 + 9c) \right\} \\ = m^2 \left\{ -0.252918 f_1^2 + 0.102124 f_1 f_2 \right\} \quad \left(\frac{1}{46} \right)$$

$$13 = m^2 \left\{ f_1^2 (-36\beta\delta - 6\alpha\gamma + 246\alpha + 54c\alpha + 16\beta\gamma) + f_1 f_2 (126 + 22c) \right\} \\ = m^2 \left\{ -0.055817 f_1^2 + 0.321522 f_1 f_2 \right\} \quad \left(\frac{1}{41} \right)$$

$$14 = m^2 \left\{ (-\alpha\gamma + 9\alpha\delta + 25\beta^2 - 8\alpha\beta + 192\alpha\delta - 22c\beta) f_1^2 + f_1 f_2 (-0.5\gamma + 4.5\delta) \right\} \\ = m^2 \left\{ 0.292988 f_1^2 + 0.302220 f_1 f_2 \right\} \quad \left(\frac{1}{40.5} \right)$$

$$15 = m^2 \left\{ (-\alpha\gamma + 9\alpha\delta + 192\alpha\delta - 24\alpha\beta + 25\beta^2 - 216c\beta) f_1^2 + f_1 f_2 (-0.5\gamma + 4.5\delta) \right\} \\ = m^2 \left\{ (0.017314 f_1^2 + 0.302220 f_1 f_2) \right\} \quad \left(\frac{1}{40.5} \right)$$

$$16 = m^2 \left\{ 0.068228 f_1^2 + 0.053586 f_1 f_2 \right\} \quad \left(\frac{1}{21.5} \right)$$

$$17 = m^2 \left\{ 0.144181 f_1^2 + 0.160258 f_1 f_2 \right\} \quad \left(\frac{1}{25.2} \right)$$

$$\begin{aligned}
18 &= m^2 \left\{ -0.007967 f_1^{*2} \right\} \left(\frac{1}{1100} \right) \\
19 &= m^2 \left\{ -0.11091 f_1^{*2} \right\} \left(\frac{1}{11,000} \right) \\
20 &= m^2 \left\{ 0.042023 f_1^{*2} \right\} \left(\frac{1}{1156} \right) \\
21 &= m^2 \left\{ 0.101340 f_1^{*2} \right\} \left(\frac{1}{1176} \right) \\
22 &= m^2 \left\{ 0.143804 f_1^{*2} \right\} \left(\frac{1}{1024} \right) \\
23 &= m^2 \left\{ -0.0020649 f_1^{*2} \right\} \left(\frac{1}{6722} \right) \\
24 &= m^2 \left\{ -0.0064067 f_1^{*2} \right\} \left(\frac{1}{6712} \right) \\
25 &= m^2 \left\{ 0.125801 f_1^{*2} \right\} \left(\frac{1}{2704} \right) \\
26 &= m^2 \left\{ 0.125801 f_1^{*2} \right\} \left(\frac{1}{2724} \right) \\
27 &= m^2 \left\{ 0.048230 f_1^{*2} \right\} \left(\frac{1}{3364} \right) \\
28 &= m^2 \left\{ 0.144691 f_1^{*2} \right\} \left(\frac{1}{3364} \right) \\
29 &= m^2 \left\{ -0.068417 f_1^{*2} \right\} \left(\frac{1}{4224} \right) \\
30 &= m^2 \left\{ -0.201251 f_1^{*2} \right\} \left(\frac{1}{4224} \right) \\
31 &= m^2 \left\{ 0.137830 f_1^{*2} \right\} \left(\frac{1}{5144} \right) \\
32 &= m^2 \left\{ 0.066848 f_1^{*2} \right\} \left(\frac{1}{8100} \right) \\
33 &= m^2 \left\{ 0.200543 f_1^{*2} \right\} \left(\frac{1}{8100} \right)
\end{aligned}$$

$$m^4 f_1^4 \left\{ \begin{aligned} &0.000592939 + 0.000116951 + 0.000594066 + 0.0016837375 + 0.0001169551 + 0.000594066 \\ &+ 0.0063331518 + 0.0004064206 + 0.00025132078 + 0.00036724356 + 0.0000432825 \\ &+ 0.0000946265 + 0.0000046170 + 0.0002146049 + 0.0000007494 + 0.0000009303 \\ &+ 0.0000083730 + 0.0000000397 + 0.0000085689 + 0.0000015276 + 0.0000088664 \\ &+ 0.0000201949 + 0.0000000064 + 0.0000117055 + 0.0000069147 + 0.0000102715 \\ &+ 0.0000036646 + 0.000005169 \end{aligned} \right\} = \underline{0.009899056 m^4 f_1^4}$$

$$m^4 f_1^3 f_2 \left\{ \begin{aligned} &0.0012251025 - 0.0000738133 - 0.0065281044 - 0.000736133 + 0.012794433 \\ &+ 0.0003891814 + 0.0000000063 - 0.000245435 - 0.000001960 - 0.0000531434 \\ &+ 0.0004427342 + 0.000021131 + 0.000000624 + 0.0000166019 \end{aligned} \right\}$$

$$= \underline{0.007267196 m^4 f_1^3 f_2}$$

$$m^4 f_1^2 f_2^2 \left\{ \begin{aligned} &0.0063241250 + 0.0006806125 + 0.0000116463 + 0.0063281250 + 0.0006806125 + 0.0000116463 \\ &+ 0.0064612659 + 0.0000931708 + 0 + 0.0004594592 + 0.000169915 + 0.0001529236 \\ &+ 0.000243423 + 0.0002243423 + 0.0000114858 \end{aligned} \right\}$$

$$= \underline{0.02169275 m^4 f_1^2 f_2^2}$$

$$\frac{+0.00703125 \text{ m}^4 f_1 f_2^3 + 0.001953125 \text{ m}^4 f_2^4}{}$$

$$- \text{m}^2 f_1^3 \left\{ 0.0035013756 + 0.0012293724 + 0.0003874180 + 0.0373498731 + 0.0032409620 - 0.0005501794 \right. \\ \left. - 0.0000739907 - 0.0004559537 \right\}$$

$$= - \frac{0.04463018 \text{ m}^2 f_1^3}{}$$

$$- \text{m}^2 f_1^2 f_2 \left\{ 0.0361719000 - 0.0003879438 + 0.0775164245 + 0.015512649 - 0.0000000000 \right. \\ \left. + 0.0000127605 \right\}$$

$$= - \frac{0.11493490 \text{ m}^2 f_1^2 f_2}{}$$

$$- \text{m}^2 f_1 f_2^2 \left\{ 0.0200955 + 0.0401910 \right\} = - \frac{0.06028650 \text{ m}^2 f_1 f_2^2}{}$$

$$+ f_1^2 \left\{ 0.0516901224 + 0.0032301330 + 0.0006381297 + 0.0550612958 + 0.0064611855 \right. \\ \left. + 0.0003018733 + 0.00000032268 + 0.0012007958 \right\} = \frac{0.11858829 f_1^2}{}$$

$$\frac{0.11732625 f_1 f_2}{}$$

$$\frac{0.06250000 f_2^2}{}$$

$$g_1 = 0.009899056 m^4 f_1^4 + 0.003267676 m^4 f_1^3 f_2 + 0.002169275 m^4 f_1^2 f_2^2 + 0.00703125 m^4 f_1 f_2^3 + 0.001953125 m^4 f_2^4,$$

$$- 0.04463018 m^2 f_1^3 - 0.11493490 m^2 f_1^2 f_2 - 0.06026650 m^2 f_1 f_2^2$$

$$+ 0.1158829 f_1^2 + 0.11732625 f_1 f_2 + 0.06250000 f_2^2$$

The constant term in $(\frac{\partial \omega}{\partial x})^2$ is

$$m^2 \left\{ 18a^2 f_1^2 + 72bf_1^2 + 162c^2 f_1^2 + \frac{1}{4} (\alpha f_1 + 0.5f_2)^2 + \beta f_1^2 + \frac{9}{4} \gamma f_1^2 + \frac{1}{4} \gamma f_1^2 + \frac{9}{4} \delta f_1^2 \right\}$$

$$K = - \frac{\sigma}{E} m^2 \left\{ f_1^2 (72a^2 + 288b^2 + 648c^2 + \alpha^2 + 4\beta^2 + 9\gamma^2 + \gamma^2 + 4\delta^2) + \alpha f_1 f_2 + \frac{1}{4} f_2^2 \right\}$$

$$K = - \frac{\sigma}{E} m^2 \left\{ 0.6520058 f_1^2 + 0.469305 f_1 f_2 + 0.250000 f_2^2 \right\}$$

$$\rho_2 = \frac{1}{12(1-v^2)} \left(\frac{t}{R} \right)^2 m^4 \left\{ 288 a^2 f_1^2 + 4608 b^2 f_1^2 + 23328 c^2 f_1^2 + 32 a^2 f_1^2 + 512 b^2 f_1^2 + 2592 c^2 f_1^2 \right. \\ \left. + 4 (a f_1 + 0.5 f_2)^2 + 64 \rho^2 f_1^2 + 2 \times 100 f_1^2 f_2^2 + 3245 f_1^2 \right\}$$

$$= \frac{1}{3(1-v^2)} \left(\frac{t}{R} \right)^2 m^4 \left\{ f_1^2 \left(32 a^2 + 1152 b^2 + 5832 c^2 + 8 f_2^2 + 128 f_2^2 + 648 c^2 + a^2 + 16 \rho^2 + 50 f_2^2 \right) \right. \\ \left. + 81 f_2^2 \right\} + a f_1 f_2 + 0.25 f_2^2$$

$$\rho_2 = \frac{1}{3(1-v^2)} \left(\frac{t}{R} \right)^2 m^4 \left\{ 1.730641 f_1^2 + 0.469305 f_1 f_2 + 0.250000 f_2^2 \right\}$$

$$\frac{\sigma}{E} m^2 \left\{ 1.3040116 f_1 + 0.469305 f_2 \right\} = \frac{1}{3(1-v^2)} \left(\frac{t}{R} \right)^2 m^4 \left\{ 3461282 f_1 + 0.469305 f_2 \right\}$$

$$+ m^4 \left\{ 0.039596224 f_1^3 + 0.021803088 f_1^2 f_2 + 0.043385500 f_1 f_2^2 + 0.00403125 f_2^3 \right\}$$

$$- m^2 \left\{ 0.13389054 f_1^2 + 0.22986980 f_1 f_2 + 0.06028650 f_2^2 \right\} + \left\{ 0.23717658 f_1 + 0.11732625 f_2 \right\}$$

7/4

$$\frac{\sigma}{E} m^2 \left\{ 0.469305 f_1 + 0.500000 f_2 \right\} = \frac{1}{3(1-\nu)} \left(\frac{f}{R} \right)^2 m^4 \left\{ 0.469305 f_1 + 0.500000 f_2 \right\} \\ + m^4 \left\{ 0.007267696 f_1^3 + 0.04336550 f_1^2 f_2 + 0.02109375 f_1 f_2^2 + 0.00781250 f_2^3 \right\} \\ - m^2 \left\{ 0.11493490 f_1^2 + 0.12057300 f_1 f_2 \right\} + \left\{ 0.11432625 f_1^2 + 0.1250000 f_2^2 \right\}$$

Substituting $g = f_1 + f_2, \quad f_1 = g - f_2$

$$\frac{\sigma}{E} m^2 \left\{ 1.3040116 g - 0.8347066 f_2 \right\} = \frac{1}{3(1-\nu)} \left(\frac{f}{R} \right)^2 m^4 \left\{ 3.461122 g - 2.991977 f_2 \right\} \\ + m^4 \left\{ 0.039596224 g^3 - 0.096985584 g^2 f_2 + 0.118567976 g f_2^2 - 0.054147386 f_2^3 \right\} \\ - m^2 \left\{ 0.13389054 g^2 - 0.03791128 g f_2 - 0.03567276 f_2^2 \right\} + \left\{ 0.13712658 g - 0.11985033 f_2 \right\}$$

$$\frac{\sigma}{E} m^2 \left\{ 0.469305 g + 0.030695 f_2 \right\} = \frac{1}{3(1-\nu)} \left(\frac{f}{R} \right)^2 m^4 \left\{ 0.469305 g + 0.030695 f_2 \right\} \\ + m^4 \left\{ 0.007267696 g^3 + 0.02158240 g^2 f_2 - 0.04387415 g f_2^2 + 0.02213655 f_2^3 \right\} \\ - m^2 \left\{ 0.11493490 g^2 - 0.10929660 g f_2 - 0.00563810 f_2^2 \right\} + \left\{ 0.11432625 g + 0.0078125 f_2 \right\}$$

$$\frac{0.6}{E_t} \gamma (s - 1562239) = \frac{1}{2.73} \gamma^2 (3.584465 s - 4146705) + (85)^2 (0.0648700 s^3 - 0.1620425 s^2 + 0.1161912 s - 0.0424373)$$

$$- (85) (0.0427608 s^2 + 0.0454167 s - 0.1604043) + (0.1435838 s - 0.2841436)$$

$$\frac{0.6}{E_t} \gamma (s + 15.21930) = \frac{1}{2.73} \gamma^2 (s + 15.21930) + (85)^2 (0.2439827 s^3 - 1.4293582 s^2 + 0.2031263 s + 0.2362713)$$

$$- (85) (-0.1836814 s^2 - 3.5607363 s + 3.7444177) + (0.2500000 s + 3.8213245)$$

$$s + 15.21930 s$$

$$- 1.562239 s - 23.885544$$

$$- 3.584465 s^2 + 4.146705 s$$

$$- 54.8039607 s + 63.4002168$$

$$- 2.584465 s^2 - 36.93019 s + 39.51468$$

$$(-0.9466905 s^2 - 13.52754 s + 14.42624) \gamma^2$$

$$\begin{aligned}
& 0.7437827 \rho^4 - 1.4293582 \rho^3 + 0.9031243 \rho^2 + 0.2267213 \rho \\
& - 1.1622788 \rho^3 + 2.2329991 \rho^2 - 1.0985859 \rho - 0.3698934 \\
& - 0.0641400 \rho^4 + 0.1420425 \rho^3 - 0.1161912 \rho^2 + 0.0474373 \rho \\
& - 0.9918167 \rho^3 + 2.1718068 \rho^2 - 1.7718121 \rho + 0.7252831 \\
& (0.679113 \rho^4 - 3.441406 \rho^3 + 4.991737 \rho^2 - 2.590859 \rho + 0.355390) (\rho^5)^2
\end{aligned}$$

$$\begin{aligned}
& -0.1836814 \rho^3 - 3.5607363 \rho^2 + 3.7444177 \rho \\
& + 0.2869542 \rho^2 + 5.5627211 \rho - 5.84962536 \\
& - 0.0427608 \rho^3 - 0.0454187 \rho^2 + 0.1604043 \rho \\
& - 0.6532827 \rho^2 - 0.6744201 \rho + 2.4510195 \\
& (+0.226442 \rho^3 + 3.972984 \rho^2 - 6.773123 \rho + 3.397206) (\rho^5)
\end{aligned}$$

$$\begin{aligned}
& 0.2500000 \rho^2 + 3.8223165 \rho \\
& - 0.3905598 \rho - 5.9713244 \\
& - 0.1435831 \rho^2 + 0.2841436 \rho \\
& - 2.1952958 \rho + 4.344357 \\
& (0.106416 \rho^2 + 1.520613 \rho - 1.627027)
\end{aligned}$$

$$\begin{aligned}
 & \left\{ 0.679113(\delta\epsilon)^2 \right\} S^4 - \left\{ 3.441406(\delta\epsilon)^2 - 0.226442(\delta\epsilon)^2 \right\} S^3 + \left\{ 4.911439(\delta\epsilon)^2 + 3.972784(\delta\epsilon)^2 \right\} S^2 + 0.106616 \\
 & \quad - 0.946691\delta^2 \left\{ S^2 \right\} \\
 & - \left\{ 2.590159(\delta\epsilon)^2 + 8.773125(\delta\epsilon)^2 - 1.520613 + 13.52754\delta^2 \right\} S + \\
 & + \left\{ 0.355390(\delta\epsilon)^2 + 3.397206(\delta\epsilon)^2 - 1.627027 + 14.47424\delta^2 \right\} = 0
 \end{aligned}$$

$$\begin{aligned}
 \frac{\sigma_R}{Et} = & \frac{\delta(1.312991S - 1.518740)}{(S - 1.562239)} + \\
 & + \frac{1}{\delta(S - 1.562239)} \left[\left\{ 0.0648700(\delta\epsilon)^2 \right\} S^3 - \left\{ 0.1420625(\delta\epsilon)^2 + 0.0424608(\delta\epsilon)^2 \right\} S^2 \right. \\
 & + \left\{ 0.1161912(\delta\epsilon)^2 - 0.0454167(\delta\epsilon)^2 + 0.1435138\delta^2 \right\} S \\
 & \left. - \left\{ 0.0474323(\delta\epsilon)^2 - 0.114043(\delta\epsilon)^2 + 0.2841436\delta^2 \right\} \right]
 \end{aligned}$$

$$\boxed{\gamma = 0.100, \quad \xi = 20,} \quad (\gamma\xi) = 2.0$$

$$(\gamma\xi)^2 = 4.0$$

719

$$\beta^2 = 0.01000$$

$$1.716452 \beta^4 - 13.312740 \beta^3 + 28.009823 \beta^2 - 26.524344 \beta + 6.733117 = 0$$

$$F(\beta) = \beta^4 - 4.900782 \beta^3 + 10.311196 \beta^2 - 9.764332 \beta + 2.471153 = 0$$

$$F'(\beta) = 4\beta^3 - 14.702346 \beta^2 + 20.622392 \beta - 9.764332$$

$$F(0.38) = 0.009279$$

$$0.02412$$

$$F'(0.38) = -2.831$$

$$F(0.382412) = 0.000033$$

$$0.000387$$

$$F'(0.382412) = -3.104$$

$$F(0.382431) = 0.K.$$

$$\boxed{\beta_1 = 0.382431}$$

$$F(\beta) = \beta^3 - 4.518351 \beta^2 + 8.513239 \beta - 6.481635 = 0$$

$$F'(\beta) = 3\beta^2 - 9.036702 \beta + 8.513239$$

$$F(1.7) = 0.035364$$

$$0.01868$$

$$F'(1.7) = 1.891$$

$$F(1.71858) = 0.000023$$

$$0.00012$$

$$F'(1.71858) = 1.913$$

$$F(1.718592) = 0.K.$$

$$\beta^2 - 2.799259 \beta + 3.771596 = 0 \quad \text{no more real root!!!}$$

$$\boxed{\beta_2 = 1.718592}$$

$$\frac{OR}{Et} = \frac{0.100 \times 1.016812}{1.179808} - \frac{1}{0.1179808} \left\{ 0.259485^3 - 0.65371165^2 + 0.51751125 - 0.1530642 \right\} \quad \underline{\underline{720}}$$

$$= 0.08618 + 0.30739 = \underline{\underline{0.39357}}$$

$$\boxed{\gamma = 0.100, \quad \xi = 30,} \quad \begin{aligned} (\gamma\xi) &= 3.0 \\ (\sqrt{\xi}) &= 9.0 \end{aligned}$$

721

$$6.112017 \xi^4 - 30.293328 \xi^3 + 56.941552 \xi^2 - 48.251762 \xi + 11.907443 = 0$$

$$F(\xi) = \xi^4 - 4.956355 \xi^3 + 9.311327 \xi^2 - 7.894572 \xi + 1.944267$$

$$F'(\xi) = 4\xi^3 - 14.869065 \xi^2 + 18.622654 \xi - 7.894572$$

$$F(0.37) =$$

$$f \quad h_2 = 0$$

722

$$\frac{\sigma_R}{E t} = \gamma \left\{ 0.97243 + 0.030365 \left(\frac{\sigma}{E} \right)^2 \right\} - 0.102675 \left(\frac{\sigma}{E} \right) + \frac{1}{\gamma} 0.181882$$

$$= \sqrt{0.707363 + 0.02209 \left(\frac{\sigma}{E} \right)^2} - 0.102675 \left(\frac{\sigma}{E} \right)$$

$$0.02209 \left(\frac{\sigma}{E} \right) = 0.102675 \left\{ 0.707363 + 0.02209 \left(\frac{\sigma}{E} \right)^2 \right\}^{\frac{1}{2}}$$

$$\left(\frac{\sigma}{E} \right)^2 = \frac{0.0105422 \times 0.707363}{0.02209 \left\{ 0.02209 - 0.0105422 \right\}} = \frac{0.0105422 \times 0.707363}{0.02209 \times 0.011549}$$

$$= 29.2290$$

$$\left(\frac{\sigma}{E} \right) = 5.4064$$

$$\left(\frac{\sigma_R}{E t} \right)_{\min} = 5.4064 \left\{ \frac{0.02209}{0.102675} - 0.102675 \right\}$$

$$= 0.5$$

723

$$\frac{w}{R} = f_0 + \frac{1}{2} f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \frac{1}{2} \cos \frac{2\pi x}{R} + f_3 \cos \frac{2\pi y}{R}$$

$$\frac{\partial w}{\partial x} = -m \left\{ \frac{1}{2} f_1 \sin \frac{\pi x}{R} \cos \frac{\pi y}{R} + 2 f_3 \sin \frac{2\pi x}{R} \right\}$$

$$\frac{\partial w}{\partial y} = -m \left\{ \frac{1}{2} f_1 \cos \frac{\pi x}{R} \sin \frac{\pi y}{R} + 2 f_3 \sin \frac{2\pi y}{R} \right\}$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{m^2}{R} \left\{ \frac{1}{2} f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + 4 f_3 \cos \frac{2\pi x}{R} \right\}$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{m^2}{R} \left\{ \frac{1}{2} f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + 4 f_3 \cos \frac{2\pi y}{R} \right\}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{m^2}{R} \left\{ \frac{1}{2} f_1 \sin \frac{\pi x}{R} \sin \frac{\pi y}{R} \right\}$$

Thus

$$\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} = \frac{1}{R} \frac{\partial^2 w}{\partial x^2}$$

$$= \frac{m^4}{R^2} \left[-\frac{1}{8} f_1^2 \cos \frac{2\pi x}{R} - \frac{1}{8} f_1^2 \cos \frac{2\pi y}{R} - (f_1 f_2 + f_1 f_3) \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} \right. \\ \left. - f_1 f_2 \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} - f_1 f_3 \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} - 16 f_2 f_3 \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} \right]$$

$$+ \frac{m^2}{R^2} \left[\frac{1}{2} f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + 4 f_3 \cos \frac{2\pi y}{R} \right]$$

$$= -\frac{m^2}{R^2} \left[\left\{ \frac{1}{8} f_1^2 m^2 - 4 f_2 \right\} \cos \frac{2\pi x}{R} + \frac{1}{8} f_1^2 m^2 \cos \frac{2\pi y}{R} + \left\{ (f_1 f_2 + f_1 f_3) m^2 - \frac{1}{2} f_1 \right\} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} \right. \\ \left. + f_1 f_2 m^2 \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} + f_1 f_3 m^2 \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} + 16 f_2 f_3 m^2 \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} \right]$$

$$\rho_1 = \frac{1}{8} \left\{ \frac{1}{8} f_1^2 m^2 - f_2 \right\}^2 + \frac{1}{512} (f_1^2 m^2)^2 + \frac{1}{4} \left\{ (f_1 f_2 + f_1 f_3) m^2 - \frac{1}{2} f_1 \right\}^2 \quad \underline{\underline{72}}$$

$$+ \frac{1}{100} f_1^2 f_2^2 m^4 + \frac{1}{100} f_1^2 f_3^2 m^4 + 4 f_2^2 f_3^2 m^4$$

$$= m^4 \left\{ \frac{1}{256} f_1^4 + \frac{1}{4} f_1^2 f_2^2 + \frac{1}{2} f_1^2 f_2 f_3 + \frac{1}{4} f_1^2 f_3^2 + \frac{1}{100} f_1^2 f_2^2 + \frac{1}{100} f_1^2 f_3^2 \right. \\ \left. + 4 f_2^2 f_3^2 \right\}$$

$$- m^2 \left\{ \frac{1}{8} f_1^2 f_2 + \frac{1}{4} f_1 f_2^2 + \frac{1}{4} f_1^2 f_3 \right\} + \left\{ 2 f_2^2 + \frac{1}{16} f_1^2 \right\}$$

$$\rho_1 = m^4 \left\{ \frac{f_1^4}{256} + \frac{13}{50} f_1^2 f_2^2 + \frac{1}{2} f_1^2 f_2 f_3 + \frac{13}{50} f_1^2 f_3^2 + 4 f_2^2 f_3^2 \right\}$$

$$- m^2 \left\{ \frac{3}{8} f_1^2 f_2 + \frac{1}{4} f_1 f_2^2 \right\} + \left\{ \frac{1}{16} f_1^2 + 2 f_2 \right\}$$

$$K = -4 \left(\frac{\sigma}{E} \right)^2 - n^2 \frac{\sigma}{E} \left[\frac{1}{4} f_1^2 + 8 f_2 \right]$$

$$\rho_2 = \frac{1}{12(1-\nu^2)} \left(\frac{f_1}{R} \right)^2 m^4 \left[f_1^2 + 32 f_2^2 + 32 f_3^2 \right]$$

$$\frac{1}{2} \rho \frac{\partial}{\partial t} m^2 = m^4 \left\{ \frac{1}{64} \rho_1^3 + \frac{13}{25} \rho_1^2 \rho_2 + \rho_1 \rho_2 \rho_3 + \frac{13}{25} \rho_1 \rho_3^2 + \frac{1}{4} \rho_1^2 \rho_2^2 + \frac{1}{4} \rho_1^2 \rho_3^2 + \frac{1}{8} \rho_1 \rho_2^3 + \frac{1}{8} \rho_1 \rho_3^3 \right\} - m^2 \left\{ \frac{3}{4} \rho_1 \rho_2 + \frac{1}{2} \rho_1 \rho_3 \right\} + \left\{ \frac{1}{8} \rho_1 \rho_2^3 + \frac{1}{8} \rho_1 \rho_3^3 \right\} + \frac{1}{6(1-v)} \left(\frac{1}{R} \right)^2 m^4 \rho_1$$

$$16 \rho_2 \frac{\partial}{\partial t} m^2 = m^4 \left\{ \frac{13}{25} \rho_1^2 \rho_2 + \frac{1}{2} \rho_1^2 \rho_3 + 8 \rho_2^2 \rho_3 + \frac{1}{4} \rho_1 \rho_2^2 \right\} - m^2 \left\{ \frac{3}{4} \rho_1^2 \right\} + \left\{ 4 \rho_2^3 + \frac{1}{6(1-v)} \left(\frac{1}{R} \right)^2 m^4 32 \rho_2 \right\}$$

$$0 = m^4 \left\{ \frac{1}{2} \rho_1^2 \rho_2 + \frac{13}{25} \rho_1^2 \rho_3 + 8 \rho_2^2 \rho_3 \right\} - m^2 \left\{ \frac{1}{4} \rho_1^2 \right\} + 0 + \frac{1}{6(1-v)} \left(\frac{1}{R} \right)^2 m^4 32 \rho_3$$

$$\frac{\partial R}{\partial t} \gamma = (\gamma E)^2 \left\{ \frac{1}{32} + \frac{26}{25} \alpha^2 + 2\alpha\beta + \frac{26}{25} \beta^2 \right\} - (\gamma E) \left\{ \frac{3}{2} \alpha + \beta \right\} + \frac{1}{4} + \frac{1}{3(1-v)} \gamma^2$$

$$\frac{\partial R}{\partial t} \gamma \alpha = (\gamma E)^2 \left\{ \frac{13}{400} \alpha + \frac{1}{32} \beta + \frac{1}{2} \alpha \beta^2 \right\} - (\gamma E) \left\{ \frac{3}{128} + \frac{1}{4} \alpha + \frac{1}{3(1-v)} \gamma^2 \alpha \right\}$$

$$0 = (\gamma E)^2 \left\{ \frac{1}{32} \alpha + \frac{13}{400} \beta + \frac{1}{2} \alpha^2 \beta \right\} - (\gamma E) \left\{ \frac{1}{64} + 0 + \frac{1}{3(1-v)} \gamma^2 \beta \right\}$$

$$0 = (\gamma E) \left\{ \left(\frac{1}{32} - \frac{13}{400} \right) \alpha + \frac{26}{25} \alpha^2 + 2\alpha^2 \beta + \left(\frac{26}{25} - \frac{1}{2} \right) \alpha \beta^2 - \frac{1}{32} \beta \right\}$$

$$- \left\{ \frac{3}{2} \alpha^2 + \alpha \beta - \frac{3}{128} \right\}$$

$$(13) \left\{ -\frac{1}{800} \alpha + \frac{26}{25} \alpha^2 + 2x^2 \beta + \frac{27}{50} \alpha \beta^2 - \frac{1}{32} \beta \right\} - \left\{ \frac{1}{2} \alpha^2 + \alpha \beta - \frac{3}{28} \right\} = 0 \quad (4)$$

But from (3),

$$\left\{ \left(\frac{1}{2} x^2 + \frac{13}{400} \right) (13)^2 + \frac{1}{3(14)} x^2 \right\} \beta + \frac{1}{32} \left\{ (13)^2 \alpha - \frac{1}{2} (13) \right\} = 0$$

$$\beta = (13) \left\{ \frac{\frac{1}{2} - (13) \alpha}{(16 \alpha^2 + \frac{26}{25}) (13)^2 + \frac{32}{3(14)} x^2} \right\}$$

$$\begin{aligned} & \left\{ \left(\frac{26}{25} x^2 - \frac{1}{800} \alpha \right) (13) - \frac{1}{2} \alpha^2 + \frac{3}{28} \right\} \left\{ (16 \alpha^2 + \frac{26}{25}) (13)^2 + \frac{32}{3(14)} x^2 \right\}^2 \\ & + \left\{ \left(2x^2 - \frac{1}{32} \right) (13) - \alpha \right\} \left\{ (16 \alpha^2 + \frac{26}{25}) (13)^2 + \frac{32}{3(14)} x^2 \right\} \left\{ \frac{1}{2} 13 - \alpha (13)^2 \right\} \\ & + \frac{27}{50} \alpha (13)^3 \left\{ \frac{1}{2} - (13) \alpha \right\}^2 = 0 \end{aligned}$$

$$\left\{ \frac{26}{25} (13) \alpha^3 - \frac{2}{3} \alpha^2 - \frac{1}{100} (13) \alpha + \frac{3}{124} \right\} \left\{ (13)^6 \alpha^4 + 2(13)^2 \left[\frac{26}{25} (13)^2 + \frac{32}{3(1-13)} \right] \alpha^2 + \left[\frac{26}{25} (13)^2 + \frac{32}{3(1-13)} \right] \alpha^2 + \frac{32}{3(1-13)} \right\}$$

$$- \left\{ 2(13)^3 \alpha^3 - 2(13)^2 \alpha^2 + \left[\frac{1}{2} (13) - \frac{1}{32} (13)^3 \right] \alpha + \frac{1}{64} (13)^2 \right\} \left\{ (13)^2 \alpha^2 + \left[\frac{26}{25} (13)^2 + \frac{32}{3(1-13)} \right] \alpha^2 + \frac{32}{3(1-13)} \right\}$$

$$+ \frac{27}{50} (13)^3 \alpha \left\{ (13)^2 \alpha^2 - (13) \alpha + \frac{1}{4} \right\} = 0$$

| α^7 | α^6 | α^5 | α^4 | α^3 | α^2 | α | 1 |
|----------------------------------|---------------------------------|--|-----------------------------------|------------------------|------------------------|-----------------------|-------------------------|
| $\frac{26}{25} (13)^5 \cdot 256$ | $-\frac{2}{3} (13)^4 \cdot 256$ | $\frac{17}{25} (13)^5 - \frac{1}{100}$ | $-\frac{1}{128} (13)^4 \cdot 256$ | $\frac{26}{25} (13)^5$ | $-\frac{1}{2} (13)^2$ | $-\frac{1}{100} (13)$ | $\frac{27}{128} (13)^2$ |
| | | $91 (13)^5$ | $91 (13)^4$ | $-\frac{1}{2} (13)^3$ | $-\frac{1}{64} (13)^4$ | $-\frac{1}{2} (13)^2$ | $-\frac{1}{4} (13)^2$ |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

177

$$A_7 x^7 + A_6 x^6 + A_5 x^5 + A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + A_0 = 0$$

$$A_7 = + 266.24 (15)^5$$

$$A_6 = - 384 (15)^4$$

$$A_5 = (15)^3 \left\{ - \frac{808}{25} (15)^2 + \frac{21632}{25} (15) + \frac{26624}{25(1-v^2)} 15^2 \right\} = (15)^3 \left\{ \frac{1432}{625} (15)^2 + \frac{26624}{25(1-v^2)} 15^2 \right\}$$

$$A_5 = (15)^3 \left\{ 229120(15)^2 + 390.09524 \right\}$$

$$A_4 = (15)^2 \left\{ 38 (15)^2 - \frac{1248}{25} (15) - \frac{512}{(1-v^2)} 15 \right\} = - (15)^2 \left\{ \frac{2984}{25} 15 - \frac{512}{(1-v^2)} 15 \right\}$$

$$A_4 = - (15)^2 \left\{ 11.92000 (15)^2 + 562.63736 15 \right\}$$

$$A_3 = (15)^3 \left\{ \frac{27}{50} (15)^2 - 8 + \frac{1}{2} (15)^2 \right\} + \left\{ \frac{26}{25} (15)^2 + \frac{32}{3(1-v^2)} 15^2 \right\} \left\{ - \left(2 + \frac{1}{25} \right) (15)^2 + \frac{676}{625} (15)^2 + \frac{832}{25(1-v^2)} 15^2 \right\}$$

$$= (15)^3 \left\{ \frac{26}{25} (15)^2 - 8 \right\} - (15) \left\{ \frac{26}{25} (15)^2 + \frac{21}{3(1-v^2)} 15^2 \right\} \left\{ \frac{599}{625} (15)^2 - \frac{832}{25(1-v^2)} 15^2 \right\}$$

$$A_3 = - (15)^3 \left\{ 8 - 104000 (15)^2 \right\} - (15) \left\{ 1.04000 (15)^2 + 11.7216117 15^2 \right\} \left\{ 0.956400 (15)^2 - 12.190476 15^2 \right\}$$

Net

$$A_3 = -(\delta^3) \left\{ -0.04326400(\delta^3)^4 + (8 - 1.4441024\delta^2)(\delta^3)^2 - 142892030\delta^4 \right\}$$

$$A_2 = -\frac{79}{100}(\delta^3)^4 + \left\{ \frac{26}{25}(\delta^3)^2 + \frac{32}{3(1-\delta^2)}\delta^2 \right\} \left\{ -\frac{11}{4}(\delta^3)^2 - \frac{28}{50}(\delta^3)^2 - \frac{96}{6(1-\delta^2)}\delta^2 \right\}$$

$$= -0.79(\delta^3)^4 + \left\{ 1.04(\delta^3)^2 + 1.7216117\delta^2 \right\} \left\{ 1.19000(\delta^3)^2 - 17.5826176\delta^2 \right\}$$

$$A_2 = \left\{ 0.4476000(\delta^3)^4 - 4.3369964\delta^2(\delta^3)^2 - 206.094272\delta^4 \right\}$$

$$A_1 = \frac{27}{200}(\delta^3)^3 - (\delta^3) \left\{ 0.52(\delta^3)^2 + 5.8606059\delta^2 \right\} \left\{ 1 - 0.0597(\delta^3)^2 + 0.029304029\delta^2 \right\}$$

$$= -(\delta^3) \left\{ 0.385(\delta^3)^2 + 5.8606059\delta^2 \right\} + (\delta^3) \left\{ 0.12(\delta^3)^2 + 5.8606059\delta^2 \right\} \left\{ 0.0599(\delta^3)^2 - 0.029304029\delta^2 \right\}$$

$$A_1 = -(\delta^3) \left\{ -0.031148(\delta^3)^4 + (0.385 - 0.3351242\delta^2)(\delta^3)^2 + (0.1717452\delta^2 + 5.8606059)\delta^4 \right\}$$

$$A_0 = 0.625 \left\{ 0.145600(\delta^3)^4 + 6.2124512\delta^2(\delta^3)^2 + 51523568\delta^4 \right\}$$

$$\boxed{\gamma = 0.1000, \quad \xi = 16}$$

730

$$(\gamma\xi) = 1.6, (\gamma\xi)^2 = 2.56, (\gamma\xi)^3 = 4.096, (\gamma\xi)^4 = 6.5536, (\gamma\xi)^5 = 10.48576$$

$$A_7 = 2791.7227$$

$$7A_7 = 19542.1009$$

$$A_6 = -2516.5824$$

$$6A_6 = -15099.4944$$

$$A_5 = +40.0032743$$

$$5A_5 = +200.0163715$$

$$A_4 = -92.5224244$$

$$4A_4 = -370.0897136$$

$$A_3 = -32.2313666$$

$$3A_3 = -96.6940998$$

$$A_2 = +240.12508$$

$$2A_2 = +480.25016$$

$$A_1 = -0.8314967$$

$$A_1 = -0.8314967$$

$$A_0 = +0.06989971$$

$$F(0.08) = +0.00054895$$

$$F'(0.08) = 1.2277$$

$$F(0.080441) = -0.00000189$$

$$F'(0.080441) = 1.236$$

$$F(0.0804456) = \text{O.K.}$$

$$\underline{\underline{\alpha_1 = +0.0804456}}$$

$$A_6' = 2791.7287$$

$$6A_6' = 16750.3722$$

731

$$A_5' = -2292.0001$$

$$5A_5' = -11460.0005$$

$$A_4' = -144.37805$$

$$4A_4' = -577.51220$$

$$A_3' = -104.137007$$

$$3A_3' = -312.411021$$

$$A_2' = -40.608731$$

$$2A_2' = -81.217462$$

$$A_1' = -0.4650389$$

$$A_1' = -0.4650389$$

$$A_0' = -0.8689070$$

$$F(0.9370) = -0.011715$$

$$F'(0.9370) = 2439$$

cccc48

$$\alpha_2 = 0.9370048$$

$$\beta_2 = - \frac{1.60000 \times 0.9992077}{38.7415950} = -0.041266$$

$$\frac{OR}{Et} = 25.6 \left\{ 0.03125 + 1.04\alpha^2 + 2\alpha\beta + 1.04\beta^2 \right\} - 16 \left\{ 1.5\alpha + \beta \right\} + 2.5$$

$$+ 0.036636$$

$$= \quad \quad \quad (-\infty)$$

$$F(\alpha) = 2791.7287 \alpha^5 - 323.86309 \alpha^4 + 159.043222 \alpha^3 - 44.924236 \alpha^2 + 1.485962 \alpha - 0.927315 = 0$$

$$F(\alpha) = 13958.6435 \alpha^4 - 1295.45236 \alpha^3 + 477.249666 \alpha^2 - 89.85472 \alpha + 1.485962$$

$$F(0.235) = + 0.018603$$

000574

$$F'(0.235) = 32.485$$

$$F(0.2344274) = + 0.000106$$

0000033

$$F'(0.2344274) = 32.117$$

$$F(0.2344241) = 0.0$$

$$\alpha_3 = - 0.2344241$$

$$\beta_3 = \frac{1.6000 \times 0.87507156}{5.0305588} = + 0.278324101$$

$$\begin{aligned} \left(\frac{OR}{Et}\right)_3 &= 25.6 \left\{ 0.03125 + 0.0571529 - 0.1304918 + 0.0805629 \right\} + 16 \times 0.0733121 \\ &\quad + 2.5 + 0.0366300 \\ &= \underline{\underline{4.6946}} \quad !!! \end{aligned}$$

$$\begin{aligned} \left(\frac{OR}{Et}\right)_3 &= 25.6 \left\{ \frac{13}{400} + \frac{1}{32} \frac{\beta}{\alpha} + \frac{1}{2} \beta^2 \right\} - 16 \frac{3}{128} \frac{1}{\alpha} + 2.5 + 0.0366300 \\ &= 25.6 \left\{ 0.0325 - 0.0371024 + 0.0382322 \right\} + 1.599648 + 2.5 + 0.0366300 \\ &= \underline{\underline{5.01036}} \quad !!! \end{aligned}$$

Contd to p. 677 !!!

733

$$\frac{w}{R} = (f_0 + \frac{1}{4}f_1) + \frac{1}{2}f_1 \left[\cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \frac{1}{4} \cos \frac{2\pi x}{R} + \frac{1}{4} \cos \frac{2\pi y}{R} \right] + \frac{1}{4}f_2 \left[\cos \frac{2\pi x}{R} + \cos \frac{2\pi y}{R} \right]$$

$$\frac{w}{R} = (f_0 + \frac{1}{4}f_1) + \frac{1}{2}f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \frac{1}{4}(\frac{1}{2}f_1 + f_2) \cos \frac{2\pi x}{R} + \frac{1}{4}(\frac{1}{2}f_1 + f_2) \cos \frac{2\pi y}{R}$$

$$\frac{\partial w}{\partial x} = -n \left[\frac{1}{2}(\frac{\pi}{R})f_1 \sin \frac{\pi y}{R} \cos \frac{\pi x}{R} + \frac{1}{2}(\frac{\pi}{R})(\frac{1}{2}f_1 + f_2) \sin \frac{2\pi x}{R} \right]$$

$$\frac{\partial w}{\partial y} = -n \left[\frac{1}{2}f_1 \cos \frac{\pi x}{R} \sin \frac{\pi y}{R} + \frac{1}{2}(\frac{1}{2}f_1 + f_2) \sin \frac{2\pi y}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x^2} = -(\frac{\pi}{R})^2 \left[\frac{1}{2}(\frac{\pi}{R})f_1 \cos \frac{\pi y}{R} \cos \frac{\pi x}{R} + (\frac{\pi}{R})^2 (\frac{1}{2}f_1 + f_2) \cos \frac{2\pi x}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial y^2} = -(\frac{\pi}{R})^2 \left[\frac{1}{2}f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + (\frac{1}{2}f_1 + f_2) \cos \frac{2\pi y}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x \partial y} = +(\frac{\pi}{R})^2 \left[\frac{1}{2}(\frac{\pi}{R}) \sin \frac{\pi y}{R} \sin \frac{\pi x}{R} \right] \quad (\mu = \frac{\pi}{R})$$

$$\Delta \Delta F = E(\frac{\pi}{R})^2 \left[n^2 \left\{ -\frac{1}{8}\mu^2 f_1^2 (\cos \frac{2\pi x}{R} + \cos \frac{2\pi y}{R}) - \frac{1}{4}\mu^2 f_1 (\frac{1}{2}f_1 + f_2) (\cos \frac{\pi x}{R} + \cos \frac{2\pi x}{R}) \cos \frac{\pi y}{R} \right. \right. \\ \left. \left. - \frac{1}{4}\mu^2 f_1 (\frac{1}{2}f_1 + f_2) \cos \frac{\pi y}{R} (\cos \frac{\pi x}{R} + \cos \frac{3\pi x}{R}) \right. \right. \\ \left. \left. - \mu^2 (\frac{1}{2}f_1 + f_2)^2 \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} + \frac{1}{2}\mu^2 f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \mu^2 (\frac{1}{2}f_1 + f_2) \cos \frac{2\pi x}{R} \right\} \right]$$

$$= -E\mu^2 (\frac{\pi}{R})^2 \left[\left\{ \frac{1}{8}f_1^2 n^2 - (\frac{1}{2}f_1 + f_2) \right\} \cos \frac{2\pi x}{R} + \frac{1}{4}f_1 (\frac{1}{2}f_1 + f_2) n^2 \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} \right. \\ \left. + \frac{1}{4}f_1 (\frac{1}{2}f_1 + f_2) n^2 \cos \frac{\pi x}{R} \cos \frac{3\pi x}{R} + \left\{ \frac{1}{2}f_1 (\frac{1}{2}f_1 + f_2) n^2 - \frac{1}{2}f_1 \right\} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} \right. \\ \left. + (\frac{1}{2}f_1 + f_2)^2 n^2 \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} + \frac{1}{8}f_1^2 n^2 \cos \frac{2\pi y}{R} \right]$$

$$F = -E\mu^2 \left(\frac{R}{n}\right)^2 \left[\frac{1}{(2\mu)^4} \left\{ \frac{1}{8} J_1^2 n^2 - \left(\frac{1}{2} J_1 + J_2\right) \right\} \cos \frac{2\pi x}{R} + \frac{1}{2^4} \frac{1}{\mu^2} J_1^2 n^2 \cos \frac{2\pi x}{R} \right. \\ \left. + \frac{1}{(\mu^2+1)^2} \left\{ \frac{1}{2} J_1 \left(\frac{1}{2} J_1 + J_2\right) n^2 - \frac{1}{2} J_1 \right\} \cos \frac{\pi x}{R} + \frac{1}{4} \frac{1}{(\mu^2+1)^2} J_1^2 \left(\frac{1}{2} J_1 + J_2\right) n^2 \cos \frac{3\pi x}{R} \cos \frac{\pi x}{R} \right. \\ \left. + \frac{1}{4} \frac{1}{(\mu^2+9)^2} J_1^2 \left(\frac{1}{2} J_1 + J_2\right) n^2 \cos \frac{2\pi x}{R} + \frac{1}{16} \frac{1}{(\mu^2+1)^2} \left(\frac{1}{2} J_1 + J_2\right)^2 n^2 \cos \frac{2\pi x}{R} \cos \frac{2\pi x}{R} \right]$$

$$Q_x = E\mu^2 \left[\frac{1}{32} J_1^2 n^2 \cos \frac{2\pi x}{R} + \frac{1}{(\mu^2+1)^2} \left\{ \frac{1}{2} J_1 \left(\frac{1}{2} J_1 + J_2\right) n^2 - \frac{1}{2} J_1 \right\} \cos \frac{\pi x}{R} \cos \frac{\pi x}{R} \right. \\ \left. + \frac{1}{4} \frac{1}{(9\mu^2+1)^2} J_1^2 \left(\frac{1}{2} J_1 + J_2\right) n^2 \cos \frac{3\pi x}{R} \cos \frac{\pi x}{R} + \frac{1}{4} \frac{1}{(\mu^2+9)^2} J_1^2 \left(\frac{1}{2} J_1 + J_2\right) n^2 \cos \frac{\pi x}{R} \cos \frac{3\pi x}{R} \right. \\ \left. + \frac{1}{4} \frac{1}{(\mu^2+1)^2} \left(\frac{1}{2} J_1 + J_2\right)^2 n^2 \cos \frac{2\pi x}{R} \cos \frac{2\pi x}{R} \right]$$

$$Q_y = E\mu^2 \left[\frac{1}{(2\mu)^4} \left\{ \frac{1}{8} J_1^2 n^2 - \left(\frac{1}{2} J_1 + J_2\right) \right\} \cos \frac{2\pi y}{R} + \frac{1}{(\mu^2+1)^2} \left\{ \frac{1}{2} J_1 \left(\frac{1}{2} J_1 + J_2\right) n^2 - \frac{1}{2} J_1 \right\} \cos \frac{\pi y}{R} \cos \frac{\pi y}{R} \right. \\ \left. + \frac{1}{4} \frac{1}{(9\mu^2+1)^2} J_1^2 \left(\frac{1}{2} J_1 + J_2\right) n^2 \cos \frac{3\pi y}{R} \cos \frac{\pi y}{R} + \frac{1}{4} \frac{1}{(\mu^2+9)^2} J_1^2 \left(\frac{1}{2} J_1 + J_2\right) n^2 \cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} \right. \\ \left. + \frac{1}{4} \frac{1}{(\mu^2+1)^2} J_1^2 \left(\frac{1}{2} J_1 + J_2\right)^2 n^2 \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} \right]$$

234

$$\begin{aligned}
\mathcal{O}_1 = & \mu^4 \left[\frac{\mathcal{O}}{(2\mu)^4} \left\{ \frac{1}{8} \rho_1^2 \rho_2^2 - \left(\frac{1}{2} \rho_1 + \rho_2 \right)^2 \right\}^2 + \frac{1}{512} \rho_1^4 \rho_2^4 + \frac{1}{4(\mu^2+1)^2} \left\{ \rho_1 \left(\frac{1}{2} \rho_1 + \rho_2 \right)^2 - \rho_1^2 \right\}^2 \right. \\
& + \frac{1}{16} \frac{1}{(9\mu^2+1)^2} \rho_1^2 \rho_2^4 \left(\frac{1}{2} \rho_1 + \rho_2 \right)^2 + \frac{1}{16} \frac{1}{(\mu^2+9)^2} \rho_1^3 \rho_2^4 \left(\frac{1}{2} \rho_1 + \rho_2 \right)^2 + \frac{1}{16} \frac{1}{(\mu^2+1)^2} \left(\frac{1}{2} \rho_1 + \rho_2 \right)^2 \rho_2^4 \left. \right] \\
= & \frac{\mu^4}{4} \left[\frac{1}{2} \rho_1^4 \rho_2^4 - \left(\frac{1}{2} \rho_1^3 \rho_2^2 + \frac{1}{2} \rho_1^2 \rho_2^3 + \frac{1}{2} \rho_1 \rho_2^4 + \frac{1}{2} \rho_1^4 \rho_2 \right) + \left(\frac{1}{2} \rho_1^2 \rho_2^2 + \rho_1 \rho_2^3 + \rho_1^2 \rho_2 \right) + \frac{1}{128} \rho_1^4 \rho_2^4 \right. \\
& + \frac{1}{(\mu^2+1)^2} \left(\frac{1}{2} \rho_1^3 \rho_2^2 + \rho_1^2 \rho_2^3 + \rho_1 \rho_2^4 + \rho_1^4 \rho_2 \right) - \left(\rho_1^3 \rho_2^2 + \rho_1^2 \rho_2^3 + \rho_1 \rho_2^4 + \rho_1^4 \rho_2 \right) + \frac{1}{(\mu^2+1)^2} \left. \right] \\
& + \frac{1}{4} \left\{ \frac{1}{(9\mu^2+1)^2} + \frac{1}{(\mu^2+9)^2} \right\} \left[\frac{1}{2} \rho_1^4 \rho_2^4 + \frac{1}{4} \rho_1^3 \rho_2^4 + \frac{1}{4} \rho_1^4 \rho_2^3 + \frac{1}{4} \rho_1^2 \rho_2^4 + \frac{1}{4} \rho_1^4 \rho_2^2 + \frac{1}{4} \rho_1^2 \rho_2^3 + \frac{1}{4} \rho_1^3 \rho_2^2 + \frac{1}{4} \rho_1^4 \rho_2 \right. \\
& + \left. \left. \frac{1}{4} \rho_1^2 \rho_2^4 + \frac{1}{4} \rho_1^4 \rho_2^3 + \frac{1}{4} \rho_1^3 \rho_2^2 + \frac{1}{4} \rho_1^4 \rho_2 \right] \right\} \\
& + 2 \rho_1^3 \rho_2^3 \rho_2^4 + \rho_2^4 \rho_2^4 \left. \right] \\
= & \frac{\mu^4}{4} \left[\rho_1^4 \left\{ \rho_2^4 + \frac{1}{128} + \frac{1}{4(\mu^2+1)^2} + \frac{1}{16(9\mu^2+1)^2} + \frac{1}{16(\mu^2+9)^2} + \frac{1}{64(\mu^2+1)^2} \right\} \right. \\
& + \rho_1^3 \rho_2 \left(\frac{1}{(\mu^2+1)^2} + \frac{1}{4(9\mu^2+1)^2} + \frac{1}{4(\mu^2+9)^2} + \frac{1}{8(\mu^2+1)^2} \right) \\
& + \rho_1^2 \rho_2^2 \left(\frac{1}{(\mu^2+1)^2} + \frac{1}{4(9\mu^2+1)^2} + \frac{1}{4(\mu^2+9)^2} + \frac{3}{8(\mu^2+1)^2} \right) + \frac{1}{2(\mu^2+1)^2} \rho_1^2 \rho_2^3 + \frac{1}{4(\mu^2+1)^2} \rho_2^4 \left. \right]
\end{aligned}$$

$$-n^2 \left\{ f_1^3 \left(\frac{1}{16\mu^4} + \frac{1}{(\mu^2+1)^2} \right) + f_1^2 f_2 \left(\frac{1}{8\mu^4} + \frac{2}{(\mu^2+1)^2} \right) \right\} + \left\{ f_1^2 \left(\frac{1}{8\mu^4} + \frac{1}{(\mu^2+1)^2} \right) \right. \\ \left. + \frac{1}{2\mu^4} f_1 f_2 + \frac{1}{2\mu^4} f_2^2 \right\}$$

$$p_1 = \frac{1}{4} \left\{ n^4 \left[\left[\frac{(1+\mu^4)}{128} + \frac{17}{64} \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{16(9\mu^2+1)^2} + \frac{\mu^4}{16(\mu^2+9)^2} \right] f_1^4 \right. \right. \\ \left. + \left[\frac{9}{8} \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{4(9\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+9)^2} \right] f_1^3 f_2 + \left[\frac{11}{8} \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{4(9\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+9)^2} \right] f_1^2 f_2^2 \right. \\ \left. + \frac{11^4}{2(\mu^2+1)^2} f_1 f_2^3 + \frac{\mu^4}{4(\mu^2+1)^2} f_2^4 \right\}$$

$$-n^2 \left[\left[\frac{1}{16} + \frac{\mu^4}{(\mu^2+1)^2} \right] f_1^3 + \left[\frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right] f_1^2 f_2 + \left[\frac{1}{8} + \frac{\mu^4}{(\mu^2+1)^2} \right] f_1^2 + \frac{1}{2} f_1 f_2 + \frac{1}{2} f_2^2 \right]$$

$$p_2 = \frac{1}{12(1-\nu^2)} \left(\frac{1}{R} \right)^2 n^4 \left\{ \frac{1}{4} (1+\mu^2)^2 f_1^2 + 2\mu^4 \left(\frac{1}{2} f_1 + f_2 \right) + 2 \left(\frac{1}{2} f_1 + f_2 \right)^2 \right\}$$

$$p_2 = \frac{1}{6(1-\nu^2)} \left(\frac{1}{R} \right)^2 n^4 \left\{ f_1^4 + \frac{1}{8} \left[\frac{1}{4} (1+\mu^2)^2 + \frac{1}{4} (1+\mu^2) \right] f_1^2 + (1+\mu^4) f_1 f_2 + (1+\mu^4) f_2^2 \right\}$$

$$\kappa = -4\left(\frac{\sigma}{2}\right)^2 - \mu^2 \frac{\sigma}{2} - \left(\frac{\sigma}{2}\right)^4 + \frac{\sigma}{2} \frac{\mu^4}{2} + \frac{\sigma}{2} \frac{\mu^4}{2}$$

$$\begin{aligned} \frac{\sigma}{2} \mu^2 \left(\lambda + \frac{3}{2} \right) &= \left(\frac{\sigma}{2} \right)^2 \left[\frac{\mu^4}{2(1+\mu^2)} + \frac{\mu^4}{4(9\mu^2+1)} + \frac{\mu^4}{4(1+\mu^2)^2} \right] \lambda^2 \\ &+ \left\{ \frac{1}{16} \frac{\mu^4}{(\mu^2+1)^2} + \frac{3}{8} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{3}{8} \frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda + \left[\frac{1+\mu^4}{64} + \frac{1}{32} \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{8(9\mu^2+1)^2} + \frac{\mu^4}{8(\mu^2+9)^2} \right] \\ &- (\sigma)^2 \left[\left(\frac{\sigma}{2} + \frac{3}{2} \frac{\mu^4}{(\mu^2+1)^2} \right) \lambda + \left(\frac{3}{32} + \frac{3}{2} \frac{\mu^4}{(1+\mu^2)^2} \right) \right] + \left[\frac{\sigma}{4} \lambda + \left(\frac{\sigma}{8} + \frac{\mu^4}{(\mu^2+1)^2} \right) \right] \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{3(1+\mu^2)} \sigma^2 \left\{ (1+\mu^4) \lambda + \left[\frac{\sigma}{4} (1+\mu^2)^2 + \frac{\sigma}{2} (1+\mu^4) \right] \right\} \\ \frac{\sigma}{2} \mu^2 \left(\lambda + \frac{\sigma}{2} \right) &= \left(\frac{\sigma}{2} \right)^2 \left[\frac{\mu^4}{4(\mu^2+1)^2} \lambda^2 + \left\{ \frac{1}{16} \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{8(9\mu^2+1)^2} + \frac{\mu^4}{8(\mu^2+9)^2} \right\} \lambda \right. \\ &+ \left. \left\{ \frac{1}{32} \frac{\mu^4}{(\mu^2+1)^2} + \frac{3}{16} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{\mu^4}{18(1+\mu^2)^2} \right\} \right] - (\sigma)^2 \left[\left(\frac{\sigma}{32} + \frac{\mu^4}{2(\mu^2+1)^2} \right) \right] + \left[\frac{\sigma}{4} + \frac{\sigma}{8} \right] \left\{ \right. \\ &+ \left. \frac{1}{3(1+\mu^2)} \sigma^2 \left[\frac{1}{2} (1+\mu^4) \lambda + \frac{1}{2} (1+\mu^4) \right] \right\} \end{aligned}$$

$$\frac{\mu^4}{4(\mu^2+1)^2} \lambda^3 \left\{ \lambda + \frac{1}{2} - \lambda - \frac{3}{2} \right\} = - \frac{\mu^4}{4(\mu^2+1)^2} \lambda^3$$

$$\left(\lambda + \frac{1}{2}\right) \left\{ \frac{11}{8} \frac{\mu^6}{(\mu^2+1)^2} + \frac{\mu^6}{4(9\mu^2+1)^2} + \frac{\mu^6}{4(\mu^2+9)^2} \right\} \lambda^2 - \left(\lambda + \frac{3}{2}\right) \frac{3\mu^4}{8(\mu^2+1)^2} \lambda^2$$

$$= \lambda^3 \left\{ \frac{\mu^6}{(\mu^2+1)^2} + \frac{\mu^6}{4(9\mu^2+1)^2} + \frac{\mu^6}{4(\mu^2+9)^2} \right\} + \lambda^2 \left\{ \frac{1}{8} \frac{\mu^6}{(\mu^2+1)^2} + \frac{\mu^6}{8(9\mu^2+1)^2} + \frac{\mu^6}{8(\mu^2+9)^2} \right\}$$

$$\left(\lambda + \frac{1}{2}\right) \left\{ \frac{27}{16} \frac{\mu^4}{(\mu^2+1)^2} + \frac{3}{8} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{3}{8} \frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda - \left(\lambda + \frac{3}{2}\right) \left\{ \frac{11}{16} \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{8(9\mu^2+1)^2} + \frac{\mu^4}{8(\mu^2+9)^2} \right\} \lambda$$

$$= \lambda^2 \left\{ \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(\mu^2+9)^2} \right\} + \lambda \left\{ -\frac{3}{16} \frac{\mu^4}{(\mu^2+1)^2} + 0 + 0 \right\}$$

$$\left(\lambda + \frac{1}{2}\right) \left\{ \frac{1+\mu^6}{64} + \frac{17}{32} \frac{\mu^6}{(\mu^2+1)^2} + \frac{\mu^6}{8(9\mu^2+1)^2} + \frac{\mu^6}{8(\mu^2+9)^2} \right\} - \left(\lambda + \frac{3}{2}\right) \left\{ \frac{9}{32} \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{16(9\mu^2+1)^2} + \frac{\mu^4}{16(\mu^2+9)^2} \right\}$$

$$= \lambda \left\{ \frac{1+\mu^6}{64} + \frac{1}{4} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(\mu^2+9)^2} \right\} + \left\{ \frac{1+\mu^6}{128} - \frac{5}{32} \frac{\mu^4}{(\mu^2+1)^2} - \frac{1}{32} \frac{\mu^4}{(9\mu^2+1)^2} - \frac{1}{32} \frac{\mu^4}{(\mu^2+9)^2} \right\}$$

$$\begin{aligned}
& (\lambda + \frac{1}{2}) \left\{ \left(\frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right) \lambda + \left(\frac{3}{32} + \frac{3\mu^4}{2(\mu^2+1)^2} \right) \right\} - \left(\lambda + \frac{3}{2} \right) \left(\frac{1}{32} + \frac{\mu^4}{2(\mu^2+1)^2} \right) \\
&= \left\{ \frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right\} \lambda^2 + \left\{ \frac{1}{32} + \frac{\mu^4}{2(\mu^2+1)^2} \right\} \left\{ 3\lambda + \frac{1}{2} - \lambda - \frac{3}{2} \right\} + \left\{ \frac{1}{16} + \frac{\mu^4}{(\mu^2+1)^2} \right\} \lambda \\
&= \left\{ \frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right\} \lambda^2 + \left\{ \frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right\} \lambda \\
&= (\lambda + \frac{1}{2}) \left[\frac{1}{4} \lambda + \left\{ \frac{1}{8} + \frac{\mu^4}{(\mu^2+1)^2} \right\} \right] - \left(\lambda + \frac{3}{2} \right) \left[\frac{1}{4} + \frac{1}{8} \right] \\
&= \frac{1}{4} \left[\lambda + \frac{1}{2} - \lambda - \frac{3}{2} \right] + \frac{1}{8} \left[\lambda + \frac{1}{2} - \lambda - \frac{3}{2} \right] + (\lambda + \frac{1}{2}) \frac{\mu^4}{(\mu^2+1)^2} \\
&= \left\{ \frac{\mu^4}{(\mu^2+1)^2} - \frac{1}{4} \right\} \lambda + \left\{ \frac{1}{2} \frac{\mu^4}{(\mu^2+1)^2} - \frac{1}{8} \right\} \\
&= (\lambda + \frac{1}{2}) \left\{ (1+\mu^4) \lambda + \frac{1}{4} (1+\mu^4)^2 + \frac{1}{2} (1+\mu^4) \right\} - \left(\lambda + \frac{3}{2} \right) \left\{ (1+\mu^4) \lambda + \frac{1}{2} (1+\mu^4) \right\} \\
&= - \left\{ (1+\mu^4) \lambda + \frac{1}{2} (1+\mu^4) \right\} + (\lambda + \frac{1}{2}) \frac{1}{4} (1+\mu^4)^2 \\
&= \left\{ \frac{1}{4} (1+\mu^4)^2 - (1+\mu^4) \right\} \lambda + \left\{ \frac{1}{8} (1+\mu^4)^2 - \frac{1}{2} (1+\mu^4) \right\}
\end{aligned}$$

$$\begin{aligned}
0 = (V_5)^2 & \left[\left\{ \frac{3\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{(9\mu^2+1)^2} + \frac{\mu^4}{(\mu^2+9)^2} + \frac{3}{2} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{3}{2} \frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda^2 \right. \\
& + \left\{ \frac{1+\mu^4}{16} + \frac{1}{4} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(\mu^2+9)^2} - \frac{5}{8} \frac{\mu^4}{(\mu^2+1)^2} - \frac{1}{8} \frac{\mu^4}{(9\mu^2+1)^2} - \frac{1}{8} \frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda \\
& - (V_5) \left[\left\{ \frac{1}{2} + \frac{9\mu^4}{(\mu^2+1)^2} \right\} \lambda^2 + \left\{ \frac{1}{2} + \frac{9\mu^4}{(\mu^2+1)^2} \right\} \lambda + \left\{ \frac{4\mu^4}{(\mu^2+1)^2} - 1 \right\} \lambda + \left\{ \frac{2\mu^4}{(\mu^2+1)^2} - \frac{1}{2} \right\} \right] \\
& - \frac{2}{3(1-\nu^2)} \gamma^2 \left[\left\{ 2(1+\mu^4) - \frac{1}{2}(1+\mu^2)^2 \right\} \lambda + \left\{ (1+\mu^4) - \frac{1}{4}(1+\mu^2)^2 \right\} \right] \\
\frac{\partial R}{\partial T} \gamma \mu^2 = (V_5)^2 & \left[\left\{ \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{4(9\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+9)^2} \right\} \lambda^2 + \left\{ \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda \right. \\
& + \left\{ \frac{1+\mu^4}{64} + \frac{1}{4} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(\mu^2+9)^2} \right\} - (V_5) \left\{ \frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right\} \left(\lambda + \frac{1}{2} \right) \\
& + \left[\frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{3(1-\nu^2)} \gamma^2 \right] \left\{ \frac{1}{4} (1+\mu^2)^2 \right\}
\end{aligned}$$

$$A_3 \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 = 0$$

741

$$A_3 = (\gamma\xi)^2 \left\{ \frac{3\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{(4\mu^2+1)^2} + \frac{\mu^4}{(\mu^2+9)^2} \right\}$$

$$A_2 = (\gamma\xi)^2 \left\{ \frac{9}{2} \frac{\mu^4}{(\mu^2+1)^2} + \frac{3}{2} \frac{\mu^4}{(4\mu^2+1)^2} + \frac{3}{2} \frac{\mu^4}{(\mu^2+9)^2} \right\} - (\gamma\xi) \left\{ \frac{1}{2} + \frac{8\mu^4}{(\mu^2+1)^2} \right\}$$

$$A_1 = (\gamma\xi)^2 \left\{ \frac{1+\mu^4}{16} + \frac{1}{4} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(4\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(\mu^2+9)^2} \right\} - (\gamma\xi) \left\{ \frac{1}{2} + \frac{8\mu^4}{(\mu^2+1)^2} \right\} + \left\{ \frac{4\mu^4}{(\mu^2+1)^2} - 1 \right\} \\ - \frac{2}{3(1-\gamma^2)} \gamma^2 \left\{ 2(1+\mu^4) - \frac{1}{2}(1+\mu^2)^2 \right\}$$

$$A_0 = (\gamma\xi)^2 \left\{ \frac{1+\mu^4}{32} - \frac{5}{8} \frac{\mu^4}{(\mu^2+1)^2} - \frac{1}{8} \frac{\mu^4}{(4\mu^2+1)^2} - \frac{1}{8} \frac{\mu^4}{(\mu^2+9)^2} \right\} + \left\{ \frac{2\mu^4}{(\mu^2+1)^2} - \frac{1}{2} \right\} \\ - \frac{2}{3(1-\gamma^2)} \gamma^2 \left\{ (1+\mu^4) - \frac{1}{4}(1+\mu^2)^2 \right\}$$

$$\begin{aligned}
 \frac{\mathcal{R}}{Et} = & \left\{ \frac{1}{\gamma} \frac{\mu^2}{(\mu^2+1)^2} + \frac{1}{12} \frac{1}{(1-\nu^2)} \frac{(\mu^2+1)^2}{\mu^2} \right\} \\
 & + \frac{1}{\gamma \mu^2} \left[(\gamma \xi)^2 \left[\frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+9)^2} \right] \lambda^2 \right. \\
 & + \left. \left[(\gamma \xi)^2 \left[\frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(\mu^2+9)^2} \right] - (\gamma \xi) \left[\frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right] \right] \lambda \right. \\
 & + \left. \left[(\gamma \xi)^2 \left[\frac{1+\mu^4}{16} + \frac{1}{4} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(\mu^2+9)^2} \right] - \frac{(\gamma \xi)}{2} \left[\frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right] \right] \right]
 \end{aligned}$$

$$\beta^2 = \left\{ \frac{\mu^2}{(\mu^2+1)^2} \right\} \sqrt{12(1-\nu^2)} \quad \text{for } \tilde{r}_{max} \text{ at } \delta/t = 0$$

$$\boxed{\mu = 1.5}$$

743

$$\mu^2 = 0.25, \quad \mu^4 = 0.0625$$

$$\frac{\mu^4}{(\mu^2+1)^2} = 0.040000, \quad \frac{\mu^6}{(9\mu^2+1)^2} = 0.005917160, \quad \frac{\mu^8}{(\mu^2+9)^2} = 0.000430460$$

$$A_3 = 0.12664762 (\gamma^5)^2$$

$$A_2 = 0.18997143 (\gamma^5)^2 - 2.8300000 (\gamma^5)$$

$$A_1 = 0.00786816 (\gamma^5)^2 - 0.8300000 (\gamma^5) - 0.140000 - 0.98443223 \gamma^2$$

$$A_0 = 0.0073721725 (\gamma^5)^2 - 0.820000 - 0.49221612 \gamma^2$$

$$\frac{GR}{Et} = \left\{ \frac{0.16}{\gamma} + 0.5723443 \gamma \right\} + \frac{6}{\gamma} \left\{ 0.04166191 (\gamma^5)^2 \lambda^2 + [0.04166191 (\gamma^5)^2 - 0.205183] \lambda \right. \\ \left. + [0.02701704 (\gamma^5)^2 - 0.1025 (\gamma^5)] \right\}$$

$$\gamma_{max} = 0.16 \sqrt{12(1-\nu^2)} = 0.52873$$

Use 23 waves

$$\underline{\underline{\gamma = 0.529}}$$

$$\boxed{\mu = 0.5}$$

$$j = 0.529, \quad j^* = 0.27984$$

$$\frac{QR}{Et} = 0.605228 \quad (\text{Absolute Min} = 0.6052275)$$

$$\boxed{\mu = 0.5 \quad \gamma = 0.529 \quad \xi = 0.7}$$

745

$$(\gamma\xi) = 0.3703, \quad (\gamma\xi)^2 = 0.13712209$$

$$0.017366186 \lambda^3 - 0.27759672 \lambda^2 - 1.40842563 \lambda - 0.55673136 = 0$$

$$F(-\lambda) = \lambda^3 + 15.98490 \lambda^2 - 81.10161 \lambda + 32.058355 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 31.96980 \lambda - 81.10161$$

$$F(0.43328) = +0.000860, \\ 0000129$$

$$F'(0.43328) = -66.68654$$

$$F(0.4332929) = 0.K$$

$$\lambda = -0.4332929$$

$$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{0.529} \{ 0.00571272 \lambda^2 - 0.04019873 \lambda - 0.03425112 \}$$

$$= \underline{\underline{0.58434}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.5864}}$$

$$\boxed{\mu = 0.5 \quad \gamma = 0.529, \quad \xi = 1.5}$$

$$(\gamma\xi) = 0.7935 \quad (\gamma\xi)^2 = 0.62964225$$

$$0.079742692 \lambda^3 - 0.53105596 \lambda^2 - 1.71699949 \lambda - 0.55310042 = 0$$

$$F(\lambda) = \lambda^3 + 6.659619 \lambda^2 - 21.531447 \lambda + 6.936064$$

$$F'(-\lambda) = 3\lambda^2 + 13.319238 \lambda - 21.531747$$

$$F(\underbrace{0.3657}_{0.000889}) = 0.001446 \quad F'(0.3657) = 16.2597$$

$$F(0.3657889) = 0.00$$

$$\lambda = -0.3657889$$

$$\frac{OR}{Et} = 0.605227 + \frac{4}{0.529} \left\{ 0.02623210 \lambda^2 - 0.13643540 \lambda - 0.06432218 \right\}$$

$$= \underline{\underline{0.52226}}$$

$$\frac{ER}{t} = \underline{\underline{0.5324}}$$

$$\underline{\Phi} =$$

$$\boxed{\mu = 0.5, \quad \beta = 0.529, \quad \xi = 2.5}$$

742

$$(\beta\xi) = 1.3225, \quad (\beta\xi)^2 = 1.74900625$$

$$0.2250748 \lambda^3 - 0.75218878 \lambda^2 - 2.06339280 \lambda - 0.54484828 = 0$$

$$F(-\lambda) = \lambda^3 + 3.3957714 \lambda^2 - 9.3152285 \lambda + 2.4597286$$

$$F'(-\lambda) = 3\lambda^2 + 6.7915428 \lambda - 9.3152285$$

$$F(0.3) = -0.0022205$$

$$F'(0.3) = 7.00247$$

$$\frac{-0.0022205}{7.00247}$$

$$F(0.2996831) = 0.0000007$$

$$0.000001$$

$$\lambda = -0.2996830$$

$$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{15.0} \left\{ 0.07216694 \lambda^2 - 0.19824556 \lambda - 0.01830328 \right\}$$

$$= \underline{\underline{0.43624}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.4641}}$$

$$\gamma^2 = 0.24841$$

$$\begin{aligned} \zeta = 0.43624^2 + 6.25 \left[0.0010931 (-\lambda)^4 - 0.0021162 (-\lambda)^3 + 0.0510580 (-\lambda)^2 \right. \\ \left. - 0.0330103 (-\lambda) + 0.0098605 \right] \end{aligned}$$

$$= 0.2184$$

$$\Phi = -0.093259$$

$$\frac{\lambda}{E} + \frac{\sigma}{E} - \frac{1}{2} n^2 \left\{ \frac{1}{16} f_1^2 + \frac{1}{8} \left(\frac{1}{2} f_1 + f_2 \right)^2 \right\} + \left(f_0 + \frac{1}{4} f_1 \right) = 0 \quad \frac{146}{(a)}$$

$$\therefore \frac{\lambda}{E} = \frac{1}{2} n^2 \left\{ \frac{3}{32} f_1^2 + \frac{1}{8} f_1 f_2 + \frac{1}{8} f_2^2 \right\} - \left(f_0 + \frac{1}{4} f_1 \right) - \nu \frac{\sigma}{E}$$

$$\varepsilon = + \left[\frac{\sigma}{E} + \nu \frac{\lambda}{E} + \frac{1}{2} n^2 \mu^2 \left\{ \frac{3}{32} f_1^2 + \frac{1}{8} f_1 f_2 + \frac{1}{8} f_2^2 \right\} \right]$$

Decrease in potential

$$= - \int \frac{\sigma}{E} \left[\frac{\sigma}{E} + \nu \frac{\lambda}{E} + n^2 \mu^2 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\} \right]$$

$$= -4 \left[2(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + n^2 \left\{ \frac{3}{32} (\mu^2 + \nu) f_1^2 + \frac{1}{8} (\mu^2 + \nu) f_1 f_2 + \frac{1}{8} (\mu^2 + \nu) f_2^2 \right\} \frac{\sigma}{E} \right. \\ \left. - 2 \left(f_0 + \frac{1}{4} f_1 \right) \frac{\sigma}{E} \right]$$

$$4 \left[\left(\frac{\lambda}{E} \right)^2 + \left(\frac{\sigma}{E} \right)^2 + 2\nu \frac{\sigma}{E} \frac{\lambda}{E} \right] = 4 \left[(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + n^4 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\}^2 \right. \\ \left. + \left(f_0 + \frac{1}{4} f_1 \right)^2 - 2n^2 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\} \left(f_0 + \frac{1}{4} f_1 \right) - \frac{2\nu \frac{\sigma}{E} n^2 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\}}{+ \frac{1}{16} f_2^2} \right. \\ \left. + \frac{2\nu \frac{\sigma}{E} \left(f_0 + \frac{1}{4} f_1 \right)}{+ \frac{1}{16} f_2^2} + \frac{2\nu \frac{\sigma}{E} n^2 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\}}{+ \frac{1}{16} f_2^2} - \frac{2\nu \frac{\sigma}{E} \left(f_0 + \frac{1}{4} f_1 \right)}{+ \frac{1}{16} f_2^2} \right]$$

$$K = -4 \left[(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + n^2 \left\{ \frac{3}{32} (\mu^2 + \nu) f_1^2 + \frac{1}{8} (\mu^2 + \nu) f_1 f_2 + \frac{1}{8} (\mu^2 + \nu) f_2^2 \right\} \frac{\sigma}{E} \right. \\ \left. - 2\nu \left(f_0 + \frac{1}{4} f_1 \right) \frac{\sigma}{E} - n^4 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\}^2 - \left(f_0 + \frac{1}{4} f_1 \right)^2 \right. \\ \left. + 2n^2 \left(f_0 + \frac{1}{4} f_1 \right) \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\} \right]$$

$$-2\sqrt{E} - 2(f_0 + \frac{1}{4}f_1) + 2\eta^2 \left\{ \frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2 \right\} = 0$$

747
(2)

$$K = -4 \left[(1-v^2) \left(\frac{\sigma}{E} \right)^2 + \eta^2 \left\{ \frac{3}{32}(\mu^2+v)f_1^2 + \frac{1}{8}(\mu^2+v)f_1f_2 + \frac{1}{8}(\mu^2+v)f_2^2 \right\} \frac{\sigma}{E} \right. \\ \left. - \eta^4 \left\{ \frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2 \right\}^2 + \left(f_0 + \frac{1}{4}f_1 \right)^2 \right]$$

$$K = -4 \left(\frac{\sigma}{E} \right)^2 - \eta^2 \mu^2 \left\{ \frac{3}{8}f_1^2 + \frac{1}{2}f_1f_2 + \frac{1}{2}f_2^2 \right\}$$

$$E = (1-v^2) \frac{\sigma}{E} - v \left(f_0 + \frac{1}{4}f_1 \right) + \eta^2 (\mu^2+v) \left\{ \frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2 \right\} \\ - v^2 \frac{\sigma}{E} - v \left(f_0 + \frac{1}{4}f_1 \right) + \eta^2 v \left\{ \frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2 \right\} = 0$$

$$E = \frac{\sigma}{E} + \eta^2 \mu^2 \left\{ \frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2 \right\}$$

$$\frac{ER}{t} = \frac{\sigma R}{Et} + \frac{1}{16} \mu^2 (115) \xi \left\{ \lambda^2 + \lambda + 0.75 \right\}$$

$$\mu = 0.5 \quad \gamma = 0.529, \quad \xi = 4$$

148

$$(\gamma\xi) = 2.116, \quad (\gamma\xi)^2 = 4.477456$$

$$0.56205915\lambda^3 - 0.88453128\lambda^2 - 2.50105775\lambda - 0.52473318 = 0$$

$$F(-\lambda) = \lambda^3 + 1.5594572\lambda^2 - 4.4105766\lambda + 0.9253597$$

$$F'(-\lambda) = 3\lambda^2 + 3.1192144\lambda - 4.4105766$$

$$F(0.232) = -0.0014491 \quad F'(0.232) = -2.525$$

0004111

$$F(0.231589) = +0.0000005$$

$$\lambda = -0.2315890$$

$$\frac{\sigma_R}{Et} = 0.605228 + \frac{4}{0.529} \left\{ 0.18653937\lambda^2 - 0.24724063\lambda - 0.09592239 \right\}$$

$$= \underline{\underline{0.36852}}$$

$$\frac{\varepsilon_R}{t} = \underline{\underline{0.4642}}$$

$$\gamma^2 = 0.279861$$

$$\bar{\gamma} = 0.36852^2 + 16 \left[0.0027986(-1)^4 - 0.0055968(\lambda)^3 + 0.0602205(-\lambda)^2 - 0.0308109(-\lambda) + 0.0060752 \right]$$

$$= 0.444$$

$$\Phi = -0.01161$$

$$\boxed{\mu = 0.5, \quad \gamma = 0.529, \quad \xi = 6}$$

249

$$(\gamma\xi) = 3.174, \quad (\gamma\xi)^2 = 10.074276$$

$$1.27588308 \lambda^3 - 0.68885538 \lambda^2 - 2.9311431 \lambda - 0.41347295 = 0$$

$$F(-\lambda) = \lambda^2 + 0.53990479 \lambda^2 - 2.29776879 \lambda + 0.37893202$$

$$F'(-\lambda) = 3\lambda^2 + 1.07980958 \lambda - 2.29776879$$

$$F(\underbrace{0.174}_{0.003137}) = 0.00073443, \quad F'(0.174) = -2.0191$$

$$F(\underbrace{0.1743637}_{11}) = 0.00000024$$

$$\lambda = -0.1743637$$

$$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{0.529} \left\{ 0.4194358 \lambda^2 - 0.23095642 \lambda - 0.05315748 \right\}$$

$$= \underline{\underline{0.60427}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.7846}} \quad \gamma^2 = 0.279841$$

$$\xi = 0.60427^2 + 36 \left[0.0061964 (-\lambda)^4 - 0.0125922 (-\lambda)^3 + 0.0105451 (-\lambda)^2 - 0.0335775 (-\lambda) + 0.0010631 \right]$$

$$= 0.5306$$

$$\Phi = -0.264410$$

$$\lambda = 0.5, \quad \gamma = 0.289, \quad \xi = 3$$

$$\gamma^2 = 0.083521, \quad \gamma\xi = 0.867, \quad (\gamma\xi)^2 = 0.751689$$

$$0.0951996228\lambda^3 - 0.56814057\lambda^2 - 1.57447779\lambda - 0.45556680 = 0$$

$$F(-\lambda) = \lambda^3 + 5.967887\lambda^2 - 16.531697\lambda + 4.785405 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 11.935774\lambda - 16.531697$$

$$F(0.331098) = +0.013445$$

$$F'(0.33) = 12.273$$

$$F(0.331098) = +0.000007$$

$$\lambda = -0.331099$$

$$\frac{OR}{Et} = 0.2190407 + \frac{4}{0.219} \left\{ 0.03131680\lambda^2 - 0.14641820\lambda - 0.06855909 \right\}$$

$$= 0.4881323$$

$$\frac{ER}{t} = \underline{\underline{0.5101}}$$

$$\begin{aligned} \bar{\xi} = 0.4881323^2 + 9 \left[0.0004678(-\lambda)^4 - 0.0009396(-\lambda)^3 + 0.037753(-\lambda)^2 \right. \\ \left. - 0.0263970(-\lambda) + 0.007446 \right] \end{aligned}$$

$$= 0.1642$$

$$\underline{\underline{\Phi}} = -0.117151$$

$$\mu = 0.5, \quad \gamma = 0.249, \quad \xi = 45$$

751

$$(\gamma\xi) = 1.3005, \quad (\gamma\xi)^2 = 1.69130025$$

$$0.214199151 \lambda^3 - 0.74511124 \lambda^2 - 1.85659407 \lambda - 0.44864183 = 0$$

$$F(-\lambda) = \lambda^3 + 3.4785912 \lambda^2 - 8.6676071 \lambda + 2.0945080 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 6.9571824 \lambda - 8.6676071$$

$$F(0.274189) = +0.0275216 \quad F'(0.27) = -6.52046$$

$$F(0.274190) = +0.0000715 \quad F(0.274190) = -6.53448$$

$$\lambda = -0.2742009$$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.249} \left\{ 0.07046280 \lambda^2 - 0.19613920 \lambda - 0.04760432 \right\}$$

$$= \underline{\underline{0.3241919}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.3746}} \quad \gamma^2 = 0.08521$$

$$\xi = 0.3241919^2 + 20.25 \left[0.0105706 (-\lambda)^4 - 0.00211412 (-\lambda)^3 + 0.0413024 (-\lambda)^2 - 0.0285434 (-\lambda) + 0.0057183 \right]$$

$$= 0.1835$$

$$\Phi = -0.044692$$

$$\mu = 0.5, \quad \gamma = 0.49, \quad \xi = 5.5$$

$$(\beta) = 1.58950, \quad (\beta)^2 = 2.52651025$$

$$0.31992648 \lambda^3 - 0.62342523 \lambda^2 - 2.02437026 \lambda - 0.44241452 = 0$$

$$F(-\lambda) = \lambda^3 + 2.5233930 \lambda^2 - 6.3391245 \lambda + 1.3624158$$

$$F'(-\lambda) = 3\lambda^2 + 5.1467860 \lambda - 6.3391245$$

$$F(0.245) = -0.0010452 \quad F'(0.245) = 4.6981$$

$$0.002435$$

$$F(0.2442665) = 0.0.$$

$$\lambda = -0.244266$$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.49} \left\{ 0.10525424 \lambda^2 - 0.12057526 \lambda - 0.09066492 \right\}$$

$$= \underline{\underline{0.2434603}}$$

$$\frac{ER}{t} = \underline{\underline{0.3207}}$$

$$\begin{aligned} \bar{\Sigma} &= 0.2434603^2 + 30.25 \left[0.00154907 (-\lambda)^4 - 0.00215816 (-\lambda)^3 + 0.0042606 (-\lambda)^2 \right. \\ &\quad \left. - 0.0222161 (-\lambda) + 0.0049465 \right] \end{aligned}$$

$$= 0.1227$$

$$\Phi = -0.016728$$

$$\boxed{\mu=0.5 \quad \gamma=0.249, \quad \xi=7}$$

151

$$(\gamma\xi)=2.023 \quad (\gamma\xi)^2=4.0929$$

$$0.51830906 \lambda^3 - 0.68139641 \lambda^2 - 2.2615845(\lambda - 0.43093956) = 0$$

$$F(-\lambda) = \lambda^3 + 1.7005229 \lambda^2 - 4.3633900 \lambda + 0.4314236 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 3.4010458 \lambda - 4.3633900$$

$$F(0.2098225) = +0.0000792$$

$$F'(0.2098) = 3.5128$$

$$F(0.2098225) = 0.00$$

$$\lambda = -0.2098225$$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.249} \left\{ 0.17050257 \lambda^2 - 0.24421243 \lambda - 0.09648944 \right\}$$

$$= 0.1925111$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.3214}}$$

$$\gamma^2 = 0.02151$$

$$\begin{aligned} \xi &= 0.1925111^2 + 49 \left[0.00255763 (-\lambda)^6 - 0.00511566 (-\lambda)^3 + 0.0698065 (-\lambda)^2 \right. \\ &\quad \left. - 0.043290 (-\lambda) + 0.0040623 \right] \end{aligned}$$

$$= 0.1222$$

$$\Phi = -0.000250$$

$$\mu = 0.5, \quad \gamma = 0.289, \quad \xi = 9$$

57

$$(15) = 2.601, \quad (15)^2 = 6.265201$$

$$0.85679661 \lambda^3 - 0.14762507 \lambda^2 - 2.52619397 \lambda - 0.41123617 = 0$$

$$F(-\lambda) = \lambda^3 + 0.9892955 \lambda^2 - 2.9492342 \lambda + 0.4299696$$

$$F'(-\lambda) = 3\lambda^2 + 1.9785910 \lambda - 2.9492342$$

$$F(0.195) = -0.0024900 \quad F'(0.195) = 2.5111$$

$$F(0.1745049) = 0.5$$

$$\lambda = -0.1745049$$

$$\frac{OR}{Et} = 0.7190407 + \frac{4}{0.249} \left\{ 0.28185120 \lambda^2 - 0.25735340 \lambda - 0.08362679 \right\}$$

$$= \underline{0.2861487}$$

$$\frac{ER}{t} = \underline{\underline{0.5022}}$$

$$\gamma^2 = 0.083521$$

$$\begin{aligned} \xi = 0.2861487^2 + 81 \left[0.00422825 (-\lambda)^6 - 0.00645650 (-\lambda)^3 + 0.0592715 (-\lambda)^2 \right. \\ \left. - 0.0247180 (-\lambda) + 0.0035902 \right] \end{aligned}$$

$$= 0.20854$$

$$\Phi = -0.041008$$

$$\boxed{\mu = 0.5, \quad \gamma = 0.196, \quad \xi = 5}$$

455

$$\gamma^2 = 0.038416$$

$$(\gamma\xi) = 0.98, \quad (\gamma\xi)^2 = 0.9604$$

$$0.12163237 \lambda^3 - 0.62115144 \lambda^2 - 1.60644129 \lambda - 0.43182874 = 0$$

$$F(-\lambda) = \lambda^3 + 5.1067938 \lambda^2 - 13.2073500 \lambda + 3.5502781$$

$$F'(-\lambda) = 3\lambda^2 + 10.2135876 \lambda - 13.2073500$$

$$F(0.307) = +0.0058663$$

$$0005993$$

$$F'(0.307) = 9.789$$

$$F(0.3075993) = +0.0000019$$

$$0000002$$

$$F'(0.3075993) = 9.78$$

$$F(0.3075995) = \text{O.K.}$$

$$\lambda = -0.3075995$$

$$\frac{QR}{Et} = 0.9285060 + \frac{4}{0.196} \left\{ 0.04001210 \lambda^2 - 0.16088291 \lambda - 0.12450263 \right\}$$

$$= \underline{\underline{0.4952826}}$$

$$\frac{ER}{t} = \underline{\underline{0.5364}}$$

$$\begin{aligned} \bar{\xi} = & 0.4952826^2 + 25 \left[0.00060025 (-\lambda)^6 - 0.0012005 (-\lambda)^5 + 0.0365201 (-\lambda)^4 \right. \\ & \left. - 0.0233636 (-\lambda) + 0.006060 \right] \end{aligned}$$

$$= 0.3014$$

$$\Phi = -0.114119$$

$$\boxed{\mu=0.5 \quad \gamma=0.196 \quad \xi=6.5}$$

251

$$\gamma^2 = 0.038416 \quad (\gamma^2) = 1.274 \quad (\gamma^2)^2 = 1.623076$$

$$0.20555871 \lambda^3 - 0.73636193 \lambda^2 - 1.29574739 \lambda - 0.42694338 = 0$$

$$F(-\lambda) = \lambda^3 + 3.5821490 \lambda^2 - 8.7311289 \lambda + 2.029900$$

$$F'(-\lambda) = 3\lambda^2 + 7.1642980 \lambda - 8.7361289$$

$$F(0.22) = -0.0009431$$

$$0.001433$$

$$F'(0.22) = 6.883$$

$$F(0.2698567) = 0.0000003$$

$$\lambda = -0.2698567$$

$$\frac{GR}{Et} = 0.9285060 + \frac{4}{0.196} \left[0.06262065 \lambda^2 - 0.19354955 \lambda - 0.08673429 \right]$$

$$= 0.3248460$$

$$\frac{\xi R}{t} = 0.3964$$

$$\gamma^2 = 0.038416$$

$$\xi = 0.3248460^2 + 42.25 \left[0.0010146225 (-\lambda)^4 - 0.002028645 (-\lambda)^3 + 0.0388669 (-\lambda)^2 \right. \\ \left. - 0.045493 (-\lambda) + 0.0069236 \right]$$

$$= 0.1462$$

$$\Phi = -0.035669$$

$$\mu = 0.5 \quad \gamma = 0.86, \quad \xi = 1.0$$

251

$$(\gamma_3) = 1.568, \quad (\gamma_3)^2 = 2.458624$$

$$0.31134888 \lambda^3 - 0.81819118 \lambda^2 - 1.97163770 \lambda - 0.4204158 = 0$$

$$F(-\lambda) = \lambda^3 + 2.6292460 \lambda^2 - 6.331957/\lambda + 1.3513556$$

$$F'(-\lambda) = 3\lambda^2 + 5.2584920 \lambda - 6.3319571$$

$$F(0.239) = 1.5018549$$

$$F'(0.239) = 4.9038$$

$$F(0.2393764) = 0.5$$

$$\lambda = -0.2393764$$

$$\frac{\delta R}{Et} = 0.9285060 + \frac{4}{0.196} \left\{ 0.10243097 \lambda^2 - 0.24900903 \lambda - 0.09429526 \right\}$$

$$= 0.1938126$$

$$\frac{\xi R}{t} = \underline{0.3051}$$

$$\gamma^2 = 0.038416$$

$$\xi = 0.1938126^2 + 6^2 \left[0.00153664 (-\lambda)^4 - 0.00307228 (-\lambda)^3 + 0.00418259 (-\lambda)^2 - 0.0201992 (-\lambda) + 0.0040502 \right]$$

$$= 0.1383$$

$$\underline{\Phi} = +2.010016$$

$$\boxed{\mu = 0.5 \quad \gamma = 0.196 \quad \xi = 10}$$

758

$$1/\xi = 0.1 \quad (1/\xi)^2 = 0.01$$

$$0.48652950 \lambda^3 - 0.17740575 \lambda^2 - 2.16511131 \lambda - 0.41051105 = 0$$

$$F(-\lambda) = \lambda^3 + 1.8033918 \lambda^2 - 4.4912205 \lambda + 0.8439119 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 3.6067836 \lambda - 4.4912205$$

$$F(0.207) = 0.0003727$$

$$F'(0.207) = 3.616$$

$$F(0.2071031) = 0.0$$

$$\lambda = -0.2071031$$

$$\frac{OR}{Et} = 0.9285060 + \frac{4}{0.196} \left\{ 0.16004839 \lambda^2 - 0.24125161 \lambda - 0.09711134 \right\}$$

$$= 0.1085248$$

$$\frac{OR}{t} = 0.2170$$

$$\gamma^2 = 0.038416$$

$$\begin{aligned} \Sigma &= 0.1085248 + 100 \left[0.0024010 (-\lambda)^2 - 0.0048020 (-\lambda) + 0.0467236 (-\lambda)^2 \right. \\ &\quad \left. - 0.0192101 (-\lambda) + 0.0032107 \right] \end{aligned}$$

$$= 0.13158$$

$$\Phi = +0.038416$$

$$\mu = 0.5, \quad \gamma = 0.196, \quad \xi = 11$$

759

$$(f_3) = 2.156 \quad (f_3)^2 = 4.648336$$

$$0.58870069 \lambda^3 - 0.88486896 \lambda^2 - 2.48285091 \lambda - 0.40464065$$

$$F(-\lambda) = \lambda^3 + 1.5030141 \lambda^2 - 3.8777786 \lambda + 0.6873453$$

$$F'(-\lambda) = 3\lambda^2 + 3.0061762 \lambda - 3.8777786$$

$$F(0.1935) = +0.0005192 \quad F'(0.1935) = 3.1838$$

163)

$$F(0.1936631) = 0.00$$

$$\lambda = -0.1936631$$

$$\frac{\sigma_R}{E t} = 0.9245060 + \frac{4}{0.196} \left\{ 0.19365456 \lambda^2 - 0.24832144 \lambda - 0.09540572 \right\}$$

$$= 0.1111228$$

$$\frac{\epsilon_R}{t} = \underline{\underline{0.3312}} \quad g^2 = 0.036616$$

$$\xi = 0.1111228^2 + 121 \left[0.00290521 (-\lambda)^4 - 0.00561062 (-\lambda)^3 + 0.0495805 (-\lambda)^2 \right. \\ \left. - 0.0190516 (-\lambda) + 0.004304 \right]$$

$$= 0.14015$$

$$\underline{\underline{\phi}} = + 0.033621$$

$$\mu = 0.5, \quad \gamma = 0.196, \quad \xi = 13$$

760

$$1/\xi = 2.568, \quad (1/\xi)^2 = 6.492304$$

$$0.62223485 \lambda^3 - 0.85600773 \lambda^2 - 2.46033572 \lambda - 0.39104661$$

$$F(-\lambda) = \lambda^3 + 1.0410745 \lambda^2 - 2.9922542 \lambda + 0.4755699$$

$$F'(-\lambda) = 3\lambda^2 + 2.0821490 \lambda - 2.9922542$$

$$F(0.1707472) = 0.0019017 \quad F'(0.17) = 2.5516$$

$$F(0.1707472) = 0.0000011$$

$$\lambda = -0.1707476$$

$$\frac{OR}{Et} = 0.922563 + \frac{L}{0.196} \left\{ 0.27248178 \lambda^2 - 0.25185822 \lambda - 0.68576716 \right\}$$

$$= 0.2167276$$

$$\frac{\xi R}{t} = 0.5316$$

$$\gamma^* = 0.038416$$

$$\xi = 0.2167276^2 + 169 \left[0.00405769 (-\lambda)^4 - 0.00811535 (-\lambda)^3 + 0.0561108 (-\lambda)^2 \right. \\ \left. - 0.1196018 (-\lambda) + 0.0024479 \right]$$

$$= 0.2047$$

$$\Phi = -0.012463$$

$$\mu = 0.5, \quad \gamma = 0.144, \quad \xi = 8$$

261

$$\gamma^2 = 0.020736 \quad (\gamma^3) = 1.152 \quad (\gamma^5)^2 = 1.327104$$

$$0.16807456 \lambda^3 - 0.69252816 \lambda^2 - 1.70144862 \lambda - 0.42042296 = 0$$

$$F(-\lambda) = \lambda^3 + 4.1203628 \lambda^2 - 10.1231776 \lambda + 2.5014075$$

$$F'(-\lambda) = 3\lambda^2 + 8.2407256 \lambda - 10.1231776$$

$$F(0.282) = -0.0032351$$

$$F'(0.282) = 2.561$$

$$0.004277$$

$$F(0.281574) = +0.0000011$$

$$\lambda = 0.281572$$

$$\frac{OR}{Et} = 1.1935287 + \frac{4}{0.144} \left\{ 0.0554969 \lambda^2 - 0.1687031 \lambda - 0.08222558 \right\}$$

$$= 0.4459175$$

$$\frac{ER}{t} = \underline{\underline{0.5248}}$$

$$\gamma^2 = 0.020736$$

$$\xi = 0.4459175^2 + 64 \left[0.00082944 (-\lambda)^4 - 0.00165888 (-\lambda)^3 - 0.031956 (-\lambda)^2 \right. \\ \left. - 0.021192 (-\lambda) + 0.0049243 \right]$$

$$= 0.3176$$

$$\Phi = -0.075218$$

$$\mu = 0.5 \quad \gamma = 0.144 \quad \xi = 10$$

762

$$(f_3) = 1.44 \quad (f_3)^2 = 2.0736$$

$$0.261650 \lambda^3 - 0.78187524 \lambda^2 - 1.82933105 \lambda - 0.41491966 = 0$$

$$F(-\lambda) = \lambda^3 + 4.8724623 \lambda^2 - 11.6371302 \lambda + 2.5192515$$

$$F'(-\lambda) = 3\lambda^2 + 9.7449246 \lambda - 11.6371302$$

$$F(0.248) = -0.001229$$

$$F'(0.248) = 9.0359$$

0002022

$$F(0.2477977) = -0.000005$$

$$\lambda = -0.2477977$$

$$\frac{\sigma R}{E t} = -1.1935287 + \frac{4}{0.144} \left\{ 0.08639014 \lambda^2 - 0.20860916 \lambda - 0.09152267 \right\}$$

$$= 0.2343567$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.3612}}$$

$$\gamma^2 = 0.020736$$

$$\xi = 0.2343567^2 + 100 \left[0.0012960 (-\lambda)^4 - 0.0025920 (-\lambda)^3 + 0.0396002 (-\lambda)^2 - 0.0198562 (-\lambda) + 0.0040316 \right]$$

$$= 0.10577$$

$$\phi = +0.014909$$

$$\mu = 0.5 \quad \gamma = 0.144 \quad \xi = 13$$

763

$$1/\xi) = 1.842$$

$$1/\xi)^2 = 3.504384$$

$$0.44382189 \lambda^3 - 0.86930716 \lambda^2 - 2.12187238 \lambda - 0.40437118 = 0$$

$$F(-\lambda) = \lambda^3 + 1.951642 \lambda^2 - 4.7609097 \lambda + 0.911125$$

$$F'(-\lambda) = 3\lambda^2 + 3.913284 \lambda - 4.7609097$$

$$F(0.2105) = +0.0008481$$

$$F'(0.2105) = 3.823$$

$$F(0.2107219) = 0.00$$

$$\lambda = -0.2107219$$

$$\frac{OR}{Et} = 1.1935247 + \frac{4}{0.144} \left\{ 0.14599933 \lambda^2 - 0.23721067 \lambda - 0.09720192 \right\}$$

$$= 0.0152115$$

$$\frac{ER}{t} = \underline{\underline{0.2122}}$$

$$\gamma^2 = 0.020736$$

$$\xi = 0.0652615^2 + 169 \left[0.00219026 (-\lambda)^4 - 0.00435048 (-\lambda)^3 + 0.0666692 (-\lambda)^2 - 0.0184940 (-\lambda) + 0.0029926 \right]$$

$$= 0.1806$$

$$\Phi = +0.071557$$

$$\mu = 0.5 \quad \gamma = 0.144 \quad \xi = 16$$

764

$$(\gamma\xi) = 2.306, \quad (\gamma\xi)^2 = 5.308416$$

$$0.67229825\lambda^3 - 0.88083242\lambda^2 - 2.33527492\lambda - 0.39107205$$

$$F(-\lambda) = \lambda^3 + 1.3101814\lambda^2 - 3.4735699\lambda + 0.5816943$$

$$F'(-\lambda) = 3\lambda^2 + 2.6203628\lambda - 3.4735699$$

$$F(0.1815) = +0.0003107 \quad F'(0.1815) = 2.899$$

1313

$$F(0.1816313) = 0.00$$

$$\lambda = -0.1816313$$

$$\frac{OR}{Et} = 1.1935287 + \frac{4}{0.144} \left\{ 0.22115875\lambda^2 - 0.25116125\lambda - 0.09274231 \right\}$$

$$= 0.0872077$$

$$\frac{\varepsilon_N}{t} = \underline{\underline{0.4336}}$$

$$\gamma^2 = 0.04736$$

$$\xi = 0.0141079^2 + 256 \left[0.00331776(-\lambda)^4 - 0.00663552(-\lambda)^3 + 0.0510560(-\lambda)^2 - 0.018243(-\lambda) + 0.0046016 \right]$$

$$= 0.1925$$

$$\underline{\Phi} = +0.061697$$

$$\mu=0.5, \quad \gamma=0.144, \quad \xi=19$$

265

$$(\delta\xi) = 2.736, \quad (\delta\xi)^2 = 7.465696$$

$$0.94804558\lambda^3 - 0.62145163\lambda^2 - 2.51953217\lambda - 0.37502077 = 0$$

$$F(-\lambda) = \lambda^3 + 0.6664685\lambda^2 - 2.6576134\lambda + 0.3955725 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 1.3329370\lambda - 2.6576134$$

$$F(0.1585) = 0.0000903, \quad F'(0.1585) = 2.3026$$

$$F(0.1585391) = 0.00$$

$$\lambda = -0.1585391$$

$$\frac{\sigma R}{Et} = 1.1935287 + \frac{4}{0.144} \left\{ 0.31166839\lambda^2 - 0.24901161\lambda - 0.07819865 \right\}$$

$$= \underline{0.3351987}$$

$$\frac{\xi R}{t} = \underline{0.8365284}$$

$$\gamma^2 = 0.020726$$

$$\xi = 0.3356987^2 + 361 \left[0.00463856(-\lambda)^4 - 0.00935712(-\lambda)^3 + 0.0587686(-\lambda)^2 - 0.0190351(-\lambda) + 0.0022712 \right]$$

$$= 0.3660$$

$$\Phi = -0.074621$$

$$\boxed{\mu = 0.5, \quad \gamma = 0.121, \quad \xi = 11}$$

766

$$\gamma^2 = 0.014641, \quad (1/3) = 1.331, \quad (1/3)^2 = 1.771561$$

$$0.22436398 \lambda^3 - 0.75487402 \lambda^2 - 1.80753056 \lambda - 0.41414629$$

$$F(-\lambda) = \lambda^3 + 3.3645063 \lambda^2 - 8.0562422 \lambda + 1.8458680$$

$$F'(-\lambda) = 3\lambda^2 + 6.7290126 \lambda - 8.0562422$$

$$F(0.26) = -0.5037384 \quad F'(0.26) = 6.1039$$

occ 6124

$$F(0.2593876) = +0.0000012$$

$$\lambda = -0.2593878$$

$$\frac{\sigma R}{E t} = 1.3915127 + \frac{4}{0.121} \left\{ 0.07380661 \lambda^2 - 0.17904639 \lambda - 0.02856517 \right\}$$

$$= \underline{0.3342549}$$

$$\frac{\varepsilon R}{t} = \underline{0.4624}$$

$$\gamma^2 = 0.014641$$

$$\begin{aligned} \bar{\varepsilon} = 0.3342549^2 + 121 \left[0.0011072 (-1)^4 - 0.0022164 (-1)^3 + 0.0382361 (-1)^2 \right. \\ \left. - 0.0200454 (-1) + 0.0042362 \right] \end{aligned}$$

$$= 0.30128$$

$$\Phi = -0.003901$$

$$\mu = 0.5, \quad \gamma = 0.121, \quad \xi = 14$$

767

$$(\beta) = 1.694, \quad (\beta)^2 = 2.869136$$

$$0.36343257 \lambda^3 - 0.14393115 \lambda^2 - 2.01946587 \lambda - 0.40605109$$

$$F(-\lambda) = \lambda^3 + 2.3221121 \lambda^2 - 5.5566454 \lambda + 1.1172117$$

$$F'(-\lambda) = 3\lambda^2 + 4.6442242 \lambda - 5.5566454$$

$$F(0.22) = 0.0178429 \quad F'(0.22) = -4.3197$$

0.04065

$$F(0.2240647) = 0.0000494 \quad F'(0.2240647) = -4.3154$$

113

$$F(0.2240760) = 0.0.$$

$$\lambda = -0.2240760$$

$$\frac{\sigma R}{Et} = 1.3915622 + \frac{4}{0.121} \left\{ 0.11955452 \lambda^2 - 0.22721547 \lambda - 0.09610593 \right\}$$

$$= \underline{0.0997487}$$

$$\frac{\varepsilon R}{t} = \underline{0.3132}$$

$$\gamma^2 = 0.014641$$

$$\begin{aligned} \zeta = & 0.0997489^2 + 196 \left[0.0017935 (-\lambda)^4 - 0.0035870 (-\lambda)^3 + 0.0421248 (-\lambda)^2 \right. \\ & \left. - 0.0146269 (-\lambda) + 0.0032360 \right] \end{aligned}$$

$$= 0.234 \quad \phi = +0.005809$$

$$\mu = 0.5, \quad \gamma = 0.121, \quad \xi = 17$$

768

$$(15) = 2.057, \quad (13)^2 = 4.231249$$

$$0.53587762 \lambda^3 - 0.68292358 \lambda^2 - 2.21082725 \lambda - 0.39601305$$

$$F(-\lambda) = \lambda^3 + 1.6676247 \lambda^2 - 4.1256196 \lambda + 0.7384790$$

$$F'(-\lambda) = 3\lambda^2 + 3.2952434 \lambda - 4.1256196$$

$$F(0.1964) = -0.0001433$$

$$F'(0.1964) = 3.313$$

$$0000426$$

$$F(0.1963574) = 0.0$$

$$\lambda = -0.1963574$$

$$\frac{GR}{Et} = 1.3915677 + \frac{4}{0.121} \left\{ 0.17628192 \lambda^2 - 0.24540308 \lambda - 0.09652118 \right\}$$

$$= 0.0182385$$

$$\frac{\Sigma R}{t} = 0.3416104$$

$$\gamma^* = 0.016661$$

$$\begin{aligned} \Sigma = 0.0182385 + 289 \left[0.1026445 (-\lambda)^4 - 0.0052871 (-\lambda)^3 + 0.0669468 (-\lambda)^2 \right. \\ \left. - 0.019470 (-\lambda) + 0.0028515 \right] \end{aligned}$$

$$= 0.23396$$

$$\Phi = +0.110746$$

$$\mu = 0.5, \quad \gamma = 0.121, \quad \xi = 20$$

269

$$(\gamma\xi) = 242, \quad (\gamma\xi)^2 = 58564$$

$$0.74169912 \lambda^3 - 0.87185132 \lambda^2 - 2.38161470 \lambda - 0.38403216 = 0$$

$$F(-\lambda) = \lambda^3 + 1.1754784 \lambda^2 - 3.2110254 \lambda + 0.5172235$$

$$F'(-\lambda) = 3\lambda^2 + 2.3509568 \lambda - 3.2110254$$

$$F(0.174) = -0.0000881, \quad F'(0.174) = 2.7111$$

0000325

$$F(0.1739675) = 0.0$$

$$\lambda = -0.1739675$$

$$\frac{GR}{Et} = 1.3915677 + \frac{4}{0.121} \left\{ 0.24391811 \lambda^2 - 0.25211119 \lambda - 0.01982241 \right\}$$

$$= \underline{0.1110133}$$

$$\frac{GR}{t} = \underline{0.5745756}$$

$$\gamma^2 = 0.016661$$

$$\xi = 0.01347069 + 400 \left\{ 0.00360015 (-\lambda)^4 - 0.0073205 (-\lambda)^3 + 0.0524019479 (-\lambda)^2 \right. \\ \left. - 0.0180354479 (-\lambda) + 0.0021922600 \right\}$$

$$= \underline{0.2615}$$

$$\phi = + 0.064063$$

$$\mu = 0.5, \quad \gamma = 0.1, \quad \xi = 13$$

770

$$(\gamma\xi) = 1.3, \quad (\gamma\xi)^2 = 1.69$$

$$0.21403648\lambda^3 - 0.76494828\lambda^2 - 1.71390913\lambda - 0.41246319 = 0$$

$$F(-\lambda) = \lambda^3 + 3.4605059\lambda^2 - 8.3346811\lambda + 1.9270876$$

$$F'(-\lambda) = 3\lambda^2 + 6.9610118\lambda - 8.3346811$$

$$F(0.262) = +0.0003017$$

$$F'(0.265) = 6.3050$$

$$0.000479$$

$$F(0.2620479) = 0.000000$$

$$\lambda = -0.2620479$$

$$\frac{OR}{Et} = 1.6572344 + 40 \left\{ 0.07040813\lambda^2 - 0.19609137\lambda - 0.01459120 \right\}$$

$$= \underline{0.4023956}$$

$$\frac{ER}{t} = \underline{0.5694}$$

$$\xi = 0.4023956^2 + 169 \left[0.00105625(-\lambda)^4 - 0.0021125(-\lambda)^3 + 0.0377215(-\lambda)^2 - 0.000090(-\lambda) + 0.0042383 \right]$$

$$= 0.443$$

$$\underline{\phi} = -0.008926$$

$$\mu = 0.5, \quad \gamma = 0.1, \quad \xi = 16$$

771

$$1/3) = .1.6, \quad 1/3) = 2.56$$

$$0.32421791 \lambda^3 - 0.82567314 \lambda^2 - 1.96191983 \lambda - 0.40604940 = 0$$

$$F(-\lambda) = \lambda^3 + 2.566611 \lambda^2 - 6.0514541 \lambda + 1.2523966$$

$$F'(-\lambda) = 3\lambda^2 + 5.0933222\lambda - 6.0514541$$

$$F(0.23) = 0.0074425$$

$$0.015774$$

$$F'(0.23) = 4.72129$$

$$F(0.2315774) = 0.5555562$$

$$17$$

$$\lambda = -0.2315771$$

$$\frac{\sigma R}{Et} = 1.6572344 + 40 \left\{ 0.10665449 \lambda^2 - 0.22136551 \lambda - 0.09481638 \right\}$$

$$= \underline{0.1429292}$$

$$\frac{\dot{\sigma} R}{t} = \underline{\underline{0.1717}}$$

$$\xi = 0.1429292^2 + 256 \left[0.0016000 (-\lambda)^4 - 0.0232000 (-\lambda)^3 + 0.0408025 (-\lambda)^2 - 0.0187025 (-\lambda) + 0.0033640 \right]$$

$$= 0.3251$$

$$\Phi = +0.107423$$

$$\boxed{\mu=0.5, \quad \gamma=0.1, \quad \xi=19}$$

772

$$(\beta) = 1.9 \quad (\beta)^2 = 3.61$$

$$0.45719791 \lambda^3 - 0.17220314 \lambda^2 - 2.12601826 \lambda - 0.39820613$$

$$F(-\lambda) = \lambda^3 + 1.907146 \lambda^2 - 4.6501050 \lambda + 0.8711952$$

$$F'(-\lambda) = 3\lambda^2 + 3.8154292 \lambda - 4.6501050$$

$$F(0.206) = +0.0029712 \quad F'(0.206) = 3.7318$$

0002951

$$F(0.2067951) = +0.0000016$$

$$\lambda = -0.2067955$$

$$\frac{\sigma R}{Et} = 1.6572344 + 4\beta \left\{ 0.15039950 \lambda^2 - 0.23910050 \lambda - 0.09721849 \right\}$$

$$= 0.0035601$$

$$\frac{\xi R}{t} = \underline{\underline{0.3341}}$$

$$\zeta = 0.0035601^2 + 3.61 \left[0.112563 (-\lambda)^2 - 0.0045125 (-\lambda)^3 + 0.0445280 (-\lambda)^2 \right. \\ \left. - 0.0149210 (-\lambda) + 0.0027154 \right]$$

$$= 0.3168$$

$$\underline{\underline{\phi}} = +0.157240$$

$$\mu = 0.5, \quad \gamma = 0.1, \quad \xi = 22$$

773

$$(\gamma\xi) = 2.2 \quad (\gamma\xi)^2 = 4.84$$

$$0.61292448\lambda^3 - 0.88453828\lambda^2 - 2.27599443\lambda - 0.38924066$$

$$F(-\lambda) = \lambda^3 + 1.4430263\lambda^2 - 3.7130330\lambda + 0.6350034$$

$$F'(-\lambda) = 3\lambda^2 + 2.8860526\lambda - 3.7130330$$

$$F(0.186) = +0.0007370$$

$$F'(0.186) = 3.072$$

$$F(0.1862399) = 0.$$

$$\lambda = -0.1862399$$

$$\frac{\sigma R}{Et} = 1.65272344 + 40 \left\{ 0.20164364\lambda^2 - 0.24935636\lambda - 0.09473253 \right\}$$

$$= \underline{\underline{0.0051000}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.6527}}$$

$$\xi = 0.0071^2 + 4.84 \left[0.002025(-\lambda)^3 - 0.00605(-\lambda)^2 + 0.0488269(-\lambda) - 0.014664(-\lambda) + 0.022103 \right]$$

$$= 0.3148$$

$$\phi = +0.155066$$

$$\mu = 0.5 \quad \gamma = 0.1 \quad \xi = 25$$

$$(\gamma\xi) = 2.5, \quad (\xi)^2 = 625$$

$$0.79154763\lambda^3 - 0.86267856\lambda^2 - 241191832\lambda - 0.37814610$$

$$F(-\lambda) = \lambda^3 + 1.0898631\lambda^2 - 30440919\lambda + 0.4786164$$

$$F'(-\lambda) = 3\lambda^2 + 2.1797262\lambda - 3.0440919$$

$$F(0.169) = -0.0003197$$

$$0.001503$$

$$F'(0.169) = 2.593$$

$$F(0.1688497) = 0.00$$

$$\lambda = -0.1688497$$

$$\frac{\sigma R}{Et} = 1.6572344 + 40 \left\{ 0.26038194\lambda^2 - 0.25211306\lambda - 0.08739350 \right\}$$

$$= 0.1612104$$

$$\frac{\sigma R}{Et} = 0.7565620$$

$$\gamma^2 = 0.51$$

$$\bar{G} = 0.0259888 + 6.45 \left\{ 0.00390625(-\lambda)^4 - 0.028125(-\lambda)^3 + 0.0538207496(-\lambda)^2 \right. \\ \left. - 0.0179325496(-\lambda) + 0.0020631058 \right\}$$

$$= 0.36137$$

$$\bar{\phi} = 1.058716$$

$$\mu = 0.5, \quad \gamma = 0.01, \quad \xi = 17$$

775

$$\gamma^2 = 0.006561, \quad (\gamma\xi) = 1.377, \quad (\gamma\xi)^2 = 1.896129$$

$$0.24014023 \lambda^3 - 0.26892966 \lambda^2 - 1.62757156 \lambda - 0.40925014$$

$$F(-\lambda) = \lambda^3 + 3.2020026 \lambda^2 - 2.6104347 \lambda + 1.7042161$$

$$F'(-\lambda) = 3\lambda^2 + 6.4040052 \lambda - 2.6104347$$

$$F(0.253) = -0.0000726$$

$$F'(0.253) = 5.798$$

$$F(0.2529875) = 0.0$$

$$\lambda = -0.2529875$$

$$\frac{\sigma R}{Et} = 2.0216645 + \frac{40}{0.01} \left\{ 0.07899136 \lambda^2 - 0.20321864 \lambda - 0.08991471 \right\}$$

$$= \underline{0.3706413}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.5760}}$$

$$\begin{aligned} \bar{\varepsilon} &= 0.14033117 + 289 \left\{ 0.001450106 (-\lambda)^4 - 0.0023701613 (-\lambda)^3 + 0.0342640805 (-\lambda)^2 \right. \\ &\quad \left. - 0.0194561174 (-\lambda) + 0.039216599 \right\} \end{aligned}$$

$$= \underline{0.5496}$$

$$\Phi = +0.061195$$

$$\mu = 0.5, \quad \gamma = 0.081, \quad \xi = 20$$

778

$$(\beta\xi) = 1.62, \quad (\beta\xi)^2 = 2.6244$$

$$0.3323740/\lambda^3 - 0.12913898\lambda^2 - 1.96992678\lambda - 0.40388191$$

$$F(-\lambda) = \lambda^3 + 24967023\lambda^2 - 5.9269880\lambda + 1.2151429$$

$$F'(-\lambda) = -3\lambda^2 + 4.9934046\lambda - 5.9269880$$

$$F(0.23) = -0.003128$$

$$F'(0.23) = 4.1198$$

$$F(0.2291727) = +0.0000023$$

$$\lambda = -0.2291732$$

$$\frac{\sigma R}{Et} = 2.0216645 + \frac{40}{0.81} \left\{ 0.10933752\lambda^2 - 0.2241248\lambda - 0.09514648 \right\}$$

$$= \underline{0.1277009}$$

$$\frac{ER}{t} = \underline{0.4180}$$

$$\xi = 0.0163075 + 400 \left\{ 0.00164025 (-\lambda)^4 - 0.0032805 (-\lambda)^3 + 0.0408631569 (-\lambda)^2 \right. \\ \left. - 0.0184666519 (-\lambda) + 0.0032652960 \right\}$$

$$= \underline{0.4661}$$

$$\Phi = +0.179671$$

$$\mu=0.5, \quad \gamma=0.081, \quad \xi=24$$

777

$$(\gamma\xi)=1.944 \quad (\gamma\xi)^2=3.779136$$

$$0.47861658\lambda^3 - 0.67615313\lambda^2 - 2.14550467\lambda - 0.39536900 = 0$$

$$F(-\lambda) = \lambda^3 + 1.8305853\lambda^2 - 4.4824108\lambda + 0.8260628$$

$$F'(-\lambda) = 3\lambda^2 + 3.6611706\lambda - 4.4824108$$

$$F(0.203) = -0.0031255$$

$$F'(0.203) = 3.116$$

$$F(0.2029653) = 0.00$$

$$\lambda = -0.2029653$$

$$\frac{\sigma R}{Et} = 2.0216685 + \frac{40}{0.81} \left\{ 0.15744602\lambda^2 - 0.24107398\lambda - 0.09715193 \right\}$$

$$= -0.0397291$$

$$\frac{\varepsilon R}{t} = \underline{0.38909}$$

$$\xi = 0.0015484 + 526 \left\{ 0.00236196(-\lambda)^4 - 0.0042239200(-\lambda)^3 + 0.0449525035(-\lambda)^2 \right. \\ \left. - 0.0126130433(-\lambda) + 0.0025655419 \right\}$$

$$= \underline{0.458245}$$

$$\Phi = + 0.264513$$

$$4 \left[\left(\frac{\lambda}{E} \right)^2 + \left(\frac{\sigma}{E} \right)^2 + 2\nu \frac{\sigma}{E} \frac{\lambda}{E} \right]$$

mf

$$= 4 \left[(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + \eta^4 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\}^2 + (f_0 + \frac{1}{4} f_1) \left\{ (f_0 + \frac{1}{4} f_1) \right. \right. \\ \left. \left. - 2\eta^2 \left(\frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right) \right\} \right]$$

$$= 4 \left[(1-\nu^2) \left(\frac{\sigma}{E} \right)^2 + \eta^4 \left\{ \frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right\}^2 + \left\{ -4 \frac{\sigma}{E} + \eta^2 \left(\frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right) \right. \right. \\ \left. \left. \left\{ \nu \frac{\sigma}{E} + \eta^2 \left(\frac{3}{64} f_1^2 + \frac{1}{16} f_1 f_2 + \frac{1}{16} f_2^2 \right) \right\} \right\} \right]$$

$$= 4 \left(\frac{\sigma}{E} \right)^2$$

$$\begin{aligned}
 \frac{\text{Elastic Strain Energy}}{\frac{1}{2} E t \text{ Area}} \left(\frac{R}{t} \right)^2 &= \left(\frac{\sigma_R}{E t} \right)^2 + \frac{1}{3} \left\{ \left(\frac{1+\mu^6}{2048} + \frac{1}{12} \frac{\mu^6}{(1+\mu^2)^2} + \frac{1}{256} \frac{\mu^6}{(9\mu^2+1)^2} + \frac{1}{256} \frac{\mu^6}{(\mu^2+9)^2} \right) \right. \\
 &+ \left[\frac{9}{128} \frac{\mu^6}{(\mu^2+1)^2} + \frac{1}{64} \frac{\mu^6}{(9\mu^2+1)^2} + \frac{1}{64} \frac{\mu^6}{(\mu^2+9)^2} \right] \lambda + \left[\frac{11}{128} \frac{\mu^6}{(\mu^2+1)^2} + \frac{1}{64} \frac{\mu^6}{(9\mu^2+1)^2} + \frac{1}{64} \frac{\mu^6}{(\mu^2+9)^2} \right] \lambda^2 \\
 &+ \frac{1}{32} \frac{\mu^6}{(\mu^2+1)^2} \lambda^3 + \frac{1}{64} \frac{\mu^6}{(\mu^2+1)^2} \lambda^4 \left\{ \right. \\
 &- (15) \left\{ \left[\frac{1}{256} + \frac{1}{16} \frac{\mu^6}{(\mu^2+1)^2} \right] + \left[\frac{1}{128} + \frac{1}{8} \frac{\mu^6}{(\mu^2+1)^2} \right] \lambda + \left[\frac{1}{128} + \frac{1}{16} \frac{\mu^6}{(\mu^2+1)^2} \right] + \frac{1}{32} \lambda + \frac{1}{32} \lambda^2 \right\} \\
 &+ \frac{1}{24(1-\nu^2)} \nu^2 \left\{ (1+\mu^6) \lambda^2 + (1+\mu^6) \lambda + \frac{1}{8} (1+\mu^6)^2 + \frac{1}{4} (1+\mu^6) \right\}
 \end{aligned}$$

$$\mu = 0.5, \quad \gamma = 0.04, \quad \xi = 27$$

280

$$(\gamma\xi) = 2.187, \quad (\gamma\xi)^2 = 4.782969$$

$$0.60575164 \lambda^3 - 0.18471254 \lambda^2 - 2.26640127 \lambda - 0.38796857$$

$$F(-\lambda) = \lambda^3 + 1.4605203 \lambda^2 - 3.7414695 \lambda + 0.6406747$$

$$F'(-\lambda) = 3\lambda^2 + 2.9210406 \lambda - 3.7414695$$

$$F(0.1863) = +0.0005963 \quad F'(0.1863) = 3.0932$$

0001928

$$F(0.1864928) = 0.0$$

$$\lambda = -0.1864928$$

$$\frac{\delta R}{Et} = 2.0216625 + \frac{40}{0.41} \left\{ 0.10926762 \lambda^2 - 0.24906738 \lambda - 0.09494584 \right\}$$

$$= -0.0309795$$

$$\frac{\delta R}{t} = \underline{0.5210242}$$

$$\begin{aligned} \bar{\epsilon} = & 0.0009597 + 729 \left\{ 0.0029893556 (-\lambda)^4 - 0.0059787113 (-\lambda)^3 \right. \\ & \left. + 0.0425074765 (-\lambda)^2 - 0.0174721533 (-\lambda) + 0.002222922 \right\} \end{aligned}$$

$$= \underline{0.4464547}$$

$$\bar{\phi} = +0.239367$$

$$\mu = 0.5, \quad \gamma = 0.081, \quad \xi = 30$$

281

$$(\gamma\xi) = 2.43, \quad (\gamma\xi)^2 = 5.9049$$

$$0.74784153 \lambda^3 - 0.87083770 \lambda^2 - 2.37807418 \lambda - 0.37969750$$

$$F(-\lambda) = \lambda^3 + 1.1644682 \lambda^2 - 3.1799173 \lambda + 0.5077245$$

$$F'(-\lambda) = 3\lambda^2 + 2.3289364 \lambda - 3.1799173$$

$$F(0.172) = 0.0003168$$

$$F'(0.172) = 2.691$$

$$F(0.1721177) = 0.0$$

$$\lambda = -0.1721177$$

$$\frac{\sigma R}{Et} = 2.0216685 + \frac{40}{0.81} \left\{ 0.24600941 \lambda^2 - 0.25214059 \lambda - 0.04954208 \right\}$$

$$= 0.1028381$$

$$\frac{\varepsilon R}{t} = 0.7948$$

$$\gamma^2 = 0.006561$$

$$\bar{\varepsilon} = 0.0165758 + 900 \left\{ 0.0036905125 (-\lambda)^4 - 0.007381125 (-\lambda)^3 \right.$$

$$\left. + 0.0524806182 (-\lambda)^2 - 0.0176551667 (-\lambda) + 0.0020217957 \right\}$$

$$= 0.41350620$$

$$\boxed{\text{for } \mu = 1}$$

782

$$\begin{aligned} \mathcal{E} &= \frac{S(\frac{R}{t})^2}{\frac{1}{2}Et \cdot A} - \left(\frac{5R}{Et}\right)^2 + \xi^2 \left[(\eta\xi)^2 \left\{ 0.00390625 \lambda^4 + 0.00781250 \lambda^3 \right. \right. \\ &\quad \left. \left. + 0.021796875 \lambda^2 + 0.017890625 \lambda + 0.005205079 \right\} \right. \\ &\quad \left. - (\eta\xi) \left\{ 0.03906250 \lambda + 0.01953125 \right\} + \left\{ 0.0312500 \lambda^2 + 0.031250 \lambda + 0.0234375 \right\} \right. \\ &\quad \left. + \eta^2 \left\{ 0.09157509 \lambda^2 + 0.09157509 \lambda + 0.04578755 \right\} \right] \\ &= \left(\frac{5R}{Et}\right)^2 + \xi^2 \left[0.00390625 (\eta\xi)^2 (-\lambda)^4 - 0.00781250 (\eta\xi)^2 (-\lambda)^3 \right. \\ &\quad \left. + \left\{ 0.021796875 (\eta\xi)^2 + 0.03125000 + 0.09157509 \eta^2 \right\} (-\lambda)^2 \right. \\ &\quad \left. - \left\{ 0.017890625 (\eta\xi)^2 - 0.03906250 (\eta\xi) + 0.031250 + 0.09157509 \eta^2 \right\} (-\lambda) \right. \\ &\quad \left. + \left\{ 0.005205079 (\eta\xi)^2 - 0.01953125 (\eta\xi) + 0.02343750 + 0.04578755 \eta^2 \right\} \right] \end{aligned}$$

$$\bar{E} = \left(\frac{6R}{Et}\right)^2 + \xi^2 \left[(\gamma\xi)^2 \left\{ 0.0012088286 + 0.002916357/\lambda + 0.003541369/\lambda^2 \right. \right. \\ \left. \left. + 0.00125/\lambda^3 + 0.000625/\lambda^4 \right\} - (\gamma\xi) \left\{ 0.00640625 + 0.0128125/\lambda \right\} \right. \\ \left. + \left\{ 0.0103125 + 0.03125/\lambda + 0.03125/\lambda^2 \right\} + \gamma^2 \left\{ 0.048649268/\lambda^2 + 0.088649268/\lambda \right. \right. \\ \left. \left. + 0.021105198 \right\} \right] \quad \underline{\underline{1/3}}$$

$$\bar{E} = \left(\frac{6R}{Et}\right)^2 + \xi^2 \left[0.000625 (\gamma\xi)^2 (-\lambda)^4 - 0.00125 (\gamma\xi)^2 (-\lambda)^3 \right. \\ \left. + \left\{ 0.003541369/(\gamma\xi)^2 + 0.03125 + 0.048649268/\lambda^2 \right\} (-\lambda)^2 \right. \\ \left. - \left\{ 0.002916357/(\gamma\xi)^2 - 0.0128125/(\gamma\xi) + 0.03125 + 0.048649268/\lambda \right\} (-\lambda) \right. \\ \left. + \left\{ 0.0012088286 (\gamma\xi)^2 - 0.00640625 (\gamma\xi) + 0.0103125 + 0.021105198/\lambda^2 \right\} \right]$$

$$\frac{\pi y}{R} = 0 \quad \frac{\pi y}{R} = 15^\circ \quad \frac{\pi y}{R} = 30^\circ \quad \frac{\pi y}{R} = 45^\circ \quad \frac{\pi y}{R} = 60^\circ \quad \frac{\pi y}{R} = 75^\circ \quad \frac{\pi y}{R} = 90^\circ$$

| $\frac{\pi x}{R}$ | $\cos \frac{\pi x}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | |
|-------------------|------------------------|---------------|---------------|---------------|---------------|---------------|--|
| 0 | 1.0000 | 0.9659 | 0.8660 | 0.7071 | 0.50000 | 0.2588 | |
| 15° | 0.9659 | 0.9330 | 0.8365 | 0.6429 | 0.4630 | 0.2500 | |
| 30° | 0.8660 | 0.8365 | 0.7500 | 0.6123 | 0.4330 | 0.2241 | |
| 45° | 0.7071 | 0.6830 | 0.6123 | 0.50000 | 0.3536 | 0.1830 | |
| 60° | 0.5000 | 0.4830 | 0.4330 | 0.3536 | 0.2500 | 0.1294 | |
| 75° | 0.2588 | 0.2500 | 0.2241 | 0.1830 | 0.1294 | 0.0670 | |
| 90° | 0 | 0 | 0 | 0 | 0 | 0 | |

$$\frac{w}{R} = \left[\cos^2 \frac{\pi(x+y)}{2R} \cos^2 \frac{\pi(x-y)}{2R} \right]$$

$$\frac{\pi y}{R} = 0 \quad \frac{\pi y}{R} = 15^\circ \quad \frac{\pi y}{R} = 30^\circ \quad \frac{\pi y}{R} = 45^\circ \quad \frac{\pi y}{R} = 60^\circ \quad \frac{\pi y}{R} = 75^\circ \quad \frac{\pi y}{R} = 90^\circ$$

| $\frac{\pi x}{R}$ | $\frac{w}{R}$ | $\cos^2 \frac{\pi x}{2R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ | $\frac{w}{R}$ |
|-------------------|---------------|---------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 0 | 1.0000 | 1.0000 | 0.9660 | 0.8704 | 0.7246 | 0.5125 | 0.3962 | 0.25000 |
| 15° | 0.9660 | 0.9829 | 0.9330 | 0.8390 | 0.6998 | 0.5373 | 0.3750 | 0.2332 |
| 30° | 0.8704 | 0.9330 | 0.8390 | 0.7500 | 0.6187 | 0.4615 | 0.3163 | 0.1875 |
| 45° | 0.7246 | 0.8536 | 0.6709 | 0.6117 | 0.50000 | 0.3643 | 0.2333 | 0.1250 |
| 60° | 0.5125 | 0.7500 | 0.5373 | 0.4665 | 0.3643 | 0.2500 | 0.1440 | 0.0625 |
| 75° | 0.3962 | 0.6295 | 0.3750 | 0.3113 | 0.2333 | 0.1440 | 0.0670 | 0.0167 |
| 90° | 0.2500 | 0.50000 | 0.2333 | 0.1875 | 0.1250 | 0.0625 | 0.0167 | 0 |
| 105° | 0.1373 | 0.3706 | 0.1250 | 0.0922 | 0.0503 | 0.0145 | 0 | 0.0167 |
| 120° | 0.0625 | 0.2500 | 0.0543 | 0.0335 | 0.0107 | 0 | 0.0145 | 0.0625 |
| 135° | 0.0215 | 0.1465 | 0.0168 | 0.0063 | 0 | 0.0107 | 0.0503 | 0.1250 |
| 150° | 0.0045 | 0.0670 | 0.0025 | 0 | 0.0063 | 0.0335 | 0.0922 | 0.1875 |
| 165° | 0.0003 | 0.0170 | 0 | 0.0025 | 0.0168 | 0.0543 | 0.1250 | 0.2332 |
| 180° | 0 | 0 | 0.0003 | 0.0045 | 0.0215 | 0.0625 | 0.1373 | 0.2500 |

85

$$\frac{w_3}{R} = \frac{1}{4} \left[\cos \frac{2\pi x}{R} + \cos \frac{2\pi y}{R} \right], \text{ !!! Not Used !!!}$$

$$\frac{\pi y}{R} = 0 \quad \frac{\pi y}{R} = 15^\circ \quad \frac{\pi y}{R} = 30^\circ \quad \frac{\pi y}{R} = 45^\circ \quad \frac{\pi y}{R} = 60^\circ \quad \frac{\pi y}{R} = 75^\circ \quad \frac{\pi y}{R} = 90^\circ$$

| $\frac{\pi x}{R}$ | $\frac{w_1}{R}$ | $\frac{w_2}{R}$ | $\frac{w_3}{R}$ | $\frac{w_4}{R}$ | $\frac{w_5}{R}$ | $\frac{w_6}{R}$ | $\frac{w_7}{R}$ |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0 | 0.5000 | 0.4665 | 0.3750 | 0.2500 | 0.1250 | 0.0335 | 0 |
| 15° | 0.4665 | 0.4330 | 0.3415 | 0.2165 | 0.0915 | 0 | -0.0335 |
| 30° | 0.3750 | 0.3415 | 0.2500 | 0.1250 | 0 | -0.0915 | -0.1250 |
| 45° | 0.2500 | 0.2165 | 0.1250 | 0 | -0.1250 | -0.2165 | -0.2500 |
| 60° | 0.1250 | 0.0915 | 0 | -0.1250 | -0.2500 | -0.3415 | -0.3750 |
| 75° | 0.0335 | 0 | -0.0915 | -0.2165 | -0.3415 | -0.4330 | -0.4665 |
| 90° | 0 | -0.0335 | -0.1250 | -0.2500 | -0.3750 | -0.4665 | -0.5000 |
| 105° | 0.0335 | 0 | -0.0915 | -0.2165 | -0.3415 | -0.4330 | -0.4665 |
| 120° | 0.1250 | 0.0915 | 0 | -0.1250 | -0.2500 | -0.3415 | -0.3750 |
| 135° | 0.2500 | 0.2165 | 0.1250 | 0 | -0.1250 | -0.2165 | -0.2500 |
| 150° | 0.3750 | 0.3415 | 0.2500 | 0.1250 | 0 | -0.0915 | -0.1250 |
| 165° | 0.4665 | 0.4330 | 0.3415 | 0.2165 | 0.0915 | 0 | -0.0335 |
| 180° | 0.5000 | 0.4665 | 0.3750 | 0.2500 | 0.1250 | 0.0335 | 0 |

$$\frac{w_6}{R} = \left(\frac{1}{6} + \frac{1}{3}f_1 \right) + \left(\frac{1}{3}f_1 + \frac{1}{2}f_2 \right) \left[\cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \frac{1}{4} \cos \frac{2\pi x}{R} + \frac{1}{4} \cos \frac{2\pi y}{R} \right]$$

$$- \frac{1}{2} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R}$$

$$= \left(\frac{1}{6} - \frac{1}{2}f_2 \right) + \left(\frac{1}{3}f_1 + \frac{1}{2}f_2 \right) \cos^2 \frac{(\pi x + \pi y)}{2R} \cos^2 \frac{(\pi x - \pi y)}{2R}$$

$$- \frac{1}{2} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R}$$

Amplitude ratio $\frac{w_1}{w_2} = \frac{f_1 + 2f_2}{-f_2} = - \frac{1 + 2\phi}{\phi}$

$$w_1 : w_2 = 1 : -\phi / (1 + 2\phi)$$

$$\mu = 1.000 ; \quad n = 26, \quad \xi = 0.5$$

186

$$\rho = -0.2667875 ; \quad \mu_1 : \mu_2 = 1 : 0.57198$$

| $\frac{\pi \xi}{R}$ | $\frac{\pi \eta}{R}$ | | | | | | |
|---------------------|----------------------|---------|---------|---------|---------|---------|---------|
| | 0 | 15° | 30° | 45° | 60° | 75° | 90° |
| 0 | 1.5720 | 1.5185 | 1.3657 | 1.1330 | 0.8485 | 0.5442 | 0.25000 |
| 15° | 1.5185 | 1.4667 | 1.3175 | 1.0905 | 0.8136 | 0.5180 | 0.2333 |
| 30° | 1.3657 | 1.3175 | 1.1790 | 0.9689 | 0.7142 | 0.4447 | 0.1875 |
| 45° | 1.1330 | 1.0905 | 0.9689 | 0.7860 | 0.5666 | 0.3380 | 0.1250 |
| 60° | 0.8485 | 0.8136 | 0.7142 | 0.5666 | 0.3930 | 0.2180 | 0.0625 |
| 75° | 0.5442 | 0.5180 | 0.4447 | 0.3380 | 0.2180 | 0.1053 | 0.0167 |
| 90° | 0.2500 | 0.2333 | 0.1875 | 0.1250 | 0.0625 | 0.0167 | 0 |
| 105° | -0.0107 | -0.0180 | -0.0360 | -0.0544 | -0.0595 | -0.0383 | 0.0167 |
| 120° | -0.2235 | -0.2220 | -0.2142 | -0.1916 | -0.1430 | -0.0595 | 0.0625 |
| 135° | -0.3829 | -0.3739 | -0.3437 | -0.2860 | -0.1916 | -0.0544 | 0.0250 |
| 150° | -0.4908 | -0.4760 | -0.4290 | -0.3439 | -0.2209 | -0.0360 | 0.0125 |
| 165° | -0.5522 | -0.5331 | -0.4760 | -0.378 | -0.2377 | -0.0180 | 0.0233 |
| 180° | -0.5740 | -0.552 | -0.4760 | -0.378 | -0.2377 | -0.0180 | 0.0233 |

$$\mu = 1.0005 \quad n = 26, \quad E = 0.000$$

253

$$S = -0.1467044; \quad w_1 = w_2 = 1.0000 = 0.20762$$

| $\frac{x}{R}$ | $\frac{1}{R}$ | | | | | | |
|---------------|---------------|---------|---------|---------|---------|---------|---------|
| | 0 | 15° | 30° | 45° | 60° | 75° | 90° |
| 0° | 1.20762 | 1.1665 | 1.0502 | 0.8754 | 0.6663 | 0.4499 | 0.2500 |
| 15° | 1.1665 | 1.1247 | 1.0127 | 0.8416 | 0.7226 | 0.4269 | 0.2333 |
| 30° | 1.0502 | 1.0197 | 0.9057 | 0.7458 | 0.5564 | 0.3628 | 0.1875 |
| 45° | 0.8754 | 0.8416 | 0.7458 | 0.6038 | 0.4377 | 0.2713 | 0.1250 |
| 60° | 0.6663 | 0.5564 | 0.4377 | 0.3628 | 0.2713 | 0.1250 | 0.0625 |
| 75° | 0.4499 | 0.3628 | 0.2713 | 0.2076 | 0.1709 | 0.0807 | 0.0167 |
| 90° | 0.2500 | 0.1875 | 0.1250 | 0.0625 | 0.0312 | 0.0156 | 0 |
| 105° | 0.0836 | 0.0731 | 0.0456 | 0.0223 | -0.0114 | -0.0139 | +0.0067 |
| 120° | -0.043 | -0.0410 | -0.0564 | -0.0627 | -0.0519 | -0.0124 | +0.0625 |
| 135° | 0.1253 | -0.1250 | -0.1208 | 0.1118 | -0.0627 | +0.0223 | -0.0625 |
| 150° | -0.1753 | -0.1712 | -0.1557 | -0.1208 | -0.0564 | +0.0457 | +0.1175 |
| 165° | -0.2002 | -0.1937 | -0.1712 | -0.1250 | -0.0468 | +0.0731 | +0.2333 |
| 180° | -0.20762 | -0.2002 | -0.1753 | -0.1253 | -0.043 | +0.0836 | +0.2500 |

$$\mu = 1.000; \quad n = 90; \quad f = 4.00$$

288

$$\rho = -0.1005343; \quad w_1: w_2 = 1: 0.12584$$

| $\frac{\pi x}{R}$ | $\frac{\pi y}{R}$ | | | | | | |
|-------------------|-------------------|---------|---------|---------|---------|---------|--------|
| | 0 | 15° | 30° | 45° | 60° | 75° | 90° |
| 0° | 1.1258 | 1.0875 | 0.9794 | 0.8176 | 0.6254 | 0.4288 | 0.2500 |
| 15° | 1.0875 | 1.0504 | 0.9443 | 0.7857 | 0.5981 | 0.4065 | 0.2333 |
| 30° | 0.9794 | 0.9443 | 0.8444 | 0.6958 | 0.5210 | 0.3445 | 0.1875 |
| 45° | 0.8176 | 0.7857 | 0.6958 | 0.5629 | 0.4088 | 0.2563 | 0.1250 |
| 60° | 0.6254 | 0.5981 | 0.5210 | 0.4088 | 0.2815 | 0.1603 | 0.0625 |
| 75° | 0.4288 | 0.4065 | 0.3445 | 0.2563 | 0.1603 | 0.0754 | 0.0167 |
| 90° | 0.2500 | 0.2333 | 0.1875 | 0.1250 | 0.0625 | 0.0167 | 0 |
| 105° | 0.1047 | 0.0935 | 0.0640 | 0.0273 | -0.0017 | -0.0084 | 0.167 |
| 120° | -0.0004 | 0.0085 | -0.0209 | -0.0351 | -0.0215 | -0.0017 | 0.0625 |
| 135° | -0.0075 | -0.0691 | -0.0208 | -0.0629 | -0.0351 | 0.0273 | 0.1250 |
| 150° | -0.0447 | -0.1028 | -0.0944 | -0.0708 | -0.0209 | 0.0640 | 0.1875 |
| 165° | -0.1012 | -0.1174 | -0.1028 | -0.0691 | -0.0065 | 0.0935 | 0.2333 |
| 180° | -0.1258 | -0.1212 | -0.1047 | -0.0675 | -0.0004 | 0.1047 | 0.2500 |

$$\mu = 1.000; \quad n = 10; \quad \xi = 16.21518$$

289

$$\beta = -0.0749409;$$

$$10_1 = 20_2 = 1:0.08815$$

| $\frac{\pi x}{R}$ | $\frac{\pi y}{R}$ | | | | | | |
|-------------------|-------------------|---------|---------|---------|---------|---------|--------|
| | 0 | 15° | 30° | 45° | 60° | 75° | 90° |
| 0 | 1.0882 | 1.0511 | 0.9467 | 0.7909 | 0.6066 | 0.4140 | 0.2500 |
| 15° | 1.0511 | 1.0152 | 0.9127 | 0.7600 | 0.5799 | 0.3970 | 0.2333 |
| 30° | 0.9467 | 0.9127 | 0.8161 | 0.6727 | 0.5047 | 0.3361 | 0.1875 |
| 45° | 0.7909 | 0.7600 | 0.6727 | 0.5441 | 0.3955 | 0.2496 | 0.1250 |
| 60° | 0.6066 | 0.5799 | 0.5047 | 0.3955 | 0.2720 | 0.1554 | 0.0615 |
| 75° | 0.4140 | 0.3970 | 0.3361 | 0.2496 | 0.1554 | 0.0729 | 0.017 |
| 90° | 0.2500 | 0.2333 | 0.1875 | 0.1250 | 0.0625 | 0.0167 | 0 |
| 105° | 0.1145 | 0.1030 | 0.0724 | 0.0342 | 0.0031 | -0.0059 | 0.0167 |
| 120° | 0.0184 | 0.0117 | -0.0047 | -0.0205 | -0.0420 | 0.0031 | 0.0625 |
| 135° | -0.0408 | -0.0434 | -0.0477 | -0.0641 | -0.0705 | 0.0342 | 0.1250 |
| 150° | -0.0718 | -0.0712 | -0.0661 | -0.0727 | -0.0677 | 0.0724 | 0.1775 |
| 165° | -0.0848 | -0.0822 | -0.0782 | -0.0737 | 0.0117 | 0.1027 | 0.222 |
| 180° | -0.0882 | -0.0848 | -0.0718 | - | 0.0737 | 0.1775 | 0.25 |



$$\mu = 2 ; \quad \kappa = 23 ; \quad \xi = 2.5$$

290

$$\rho = -0.2996130 ; \quad w_1, w_2 = 1 - 0.24602$$

| $\frac{\pi \kappa}{R}$ | η_{θ}/R | | | | | | |
|------------------------|-------------------|---------|---------|---------|---------|---------|--------|
| | 0 | 15° | 30° | 45° | 60° | 75° | 90° |
| 0 | 1.74802 | 1.6885 | 1.5182 | 1.2575 | 0.9365 | 0.5838 | 0.2500 |
| 15° | 1.6885 | 1.6309 | 1.4647 | 1.2107 | 0.8986 | 0.5620 | 0.2333 |
| 30° | 1.5182 | 1.4647 | 1.3110 | 1.0767 | 0.7904 | 0.4839 | 0.1875 |
| 45° | 1.2575 | 1.2107 | 1.0767 | 0.8740 | 0.6288 | 0.3702 | 0.1250 |
| 60° | 0.9365 | 0.8986 | 0.7904 | 0.6288 | 0.4370 | 0.2408 | 0.0625 |
| 75° | 0.5838 | 0.5620 | 0.4839 | 0.3702 | 0.2408 | 0.1171 | 0.0167 |
| 90° | 0.2500 | 0.2333 | 0.1875 | 0.1250 | 0.0625 | 0.0167 | 0 |
| 105° | -0.0563 | -0.0620 | -0.0754 | -0.0866 | -0.0823 | -0.0501 | 0.0167 |
| 120° | -0.3115 | -0.3070 | -0.2904 | -0.2538 | -0.1870 | -0.0823 | 0.0625 |
| 135° | -0.5074 | -0.4941 | -0.4517 | -0.3740 | -0.2538 | -0.0866 | 0.1250 |
| 150° | -0.6433 | -0.6232 | -0.5610 | -0.4517 | -0.2904 | -0.0754 | 0.1875 |
| 165° | -0.7222 | -0.6979 | -0.6232 | -0.4941 | -0.3070 | -0.0620 | 0.2333 |
| 180° | -0.7480 | -0.7222 | -0.6433 | -0.5074 | -0.3115 | -0.0563 | 0.2500 |

$$\mu=2, n=17, \xi=5.5$$

791

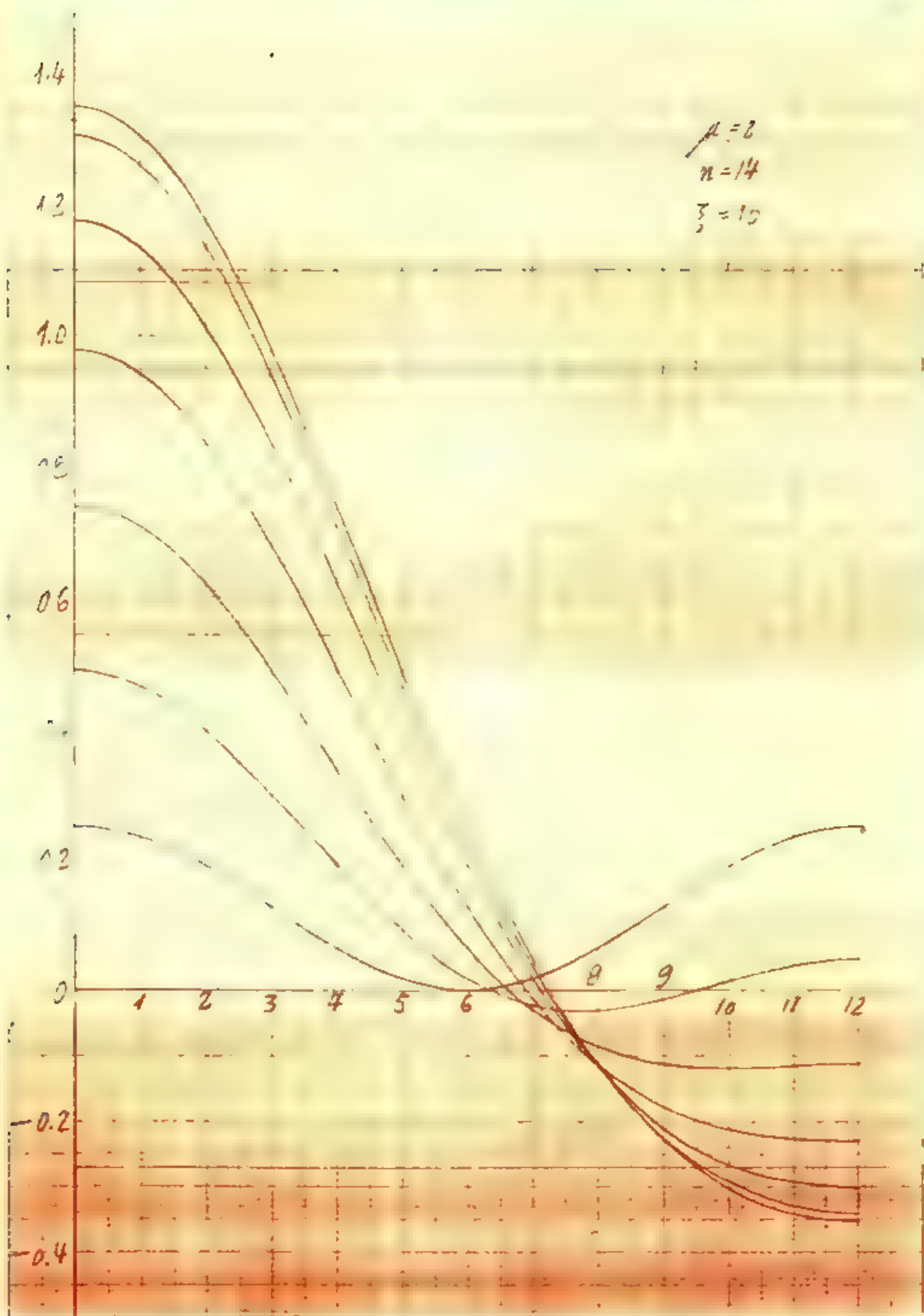
$$\rho = -0.2442866; \quad \omega_1 : \omega_2 = 1 : 0.47957$$

| $\frac{\pi x}{R}$ | $\frac{\pi y}{R}$ | | | | | | |
|-------------------|-------------------|------------|------------|------------|------------|------------|------------|
| | 0° | 15° | 30° | 45° | 60° | 75° | 90° |
| 0° | 1.4796 | 1.4292 | 1.2857 | 1.0677 | 0.8023 | 0.5203 | 0.2500 |
| 15° | 1.4292 | 1.3804 | 1.2402 | 1.0273 | 0.7689 | 0.4949 | 0.2333 |
| 30° | 1.2857 | 1.2402 | 1.1097 | 0.9123 | 0.6742 | 0.4238 | 0.1875 |
| 45° | 1.0677 | 1.0273 | 0.9123 | 0.7398 | 0.5339 | 0.3211 | 0.1250 |
| 60° | 0.8023 | 0.7689 | 0.6742 | 0.5339 | 0.3699 | 0.2061 | 0.0625 |
| 75° | 0.5203 | 0.4949 | 0.4238 | 0.3211 | 0.2061 | 0.0991 | 0.0167 |
| 90° | 0.2500 | 0.2333 | 0.1875 | 0.1250 | 0.0625 | 0.0167 | 0 |
| 105° | 0.0132 | 0.0051 | -0.0153 | -0.0375 | -0.0426 | -0.0321 | 0.0167 |
| 120° | -0.1773 | -0.1773 | -0.1742 | -0.1519 | -0.1199 | -0.0426 | 0.0625 |
| 135° | -0.3126 | -0.3107 | -0.2873 | -0.2398 | -0.1589 | -0.0375 | 0.1250 |
| 150° | -0.4108 | -0.3987 | -0.3597 | -0.2873 | -0.1742 | -0.0153 | 0.1875 |
| 165° | -0.4629 | -0.4474 | -0.3927 | -0.3107 | -0.1233 | 0.0051 | 0.2333 |
| 180° | -0.4726 | -0.4629 | -0.4108 | -0.3126 | -0.1773 | 0.0132 | 0.2500 |

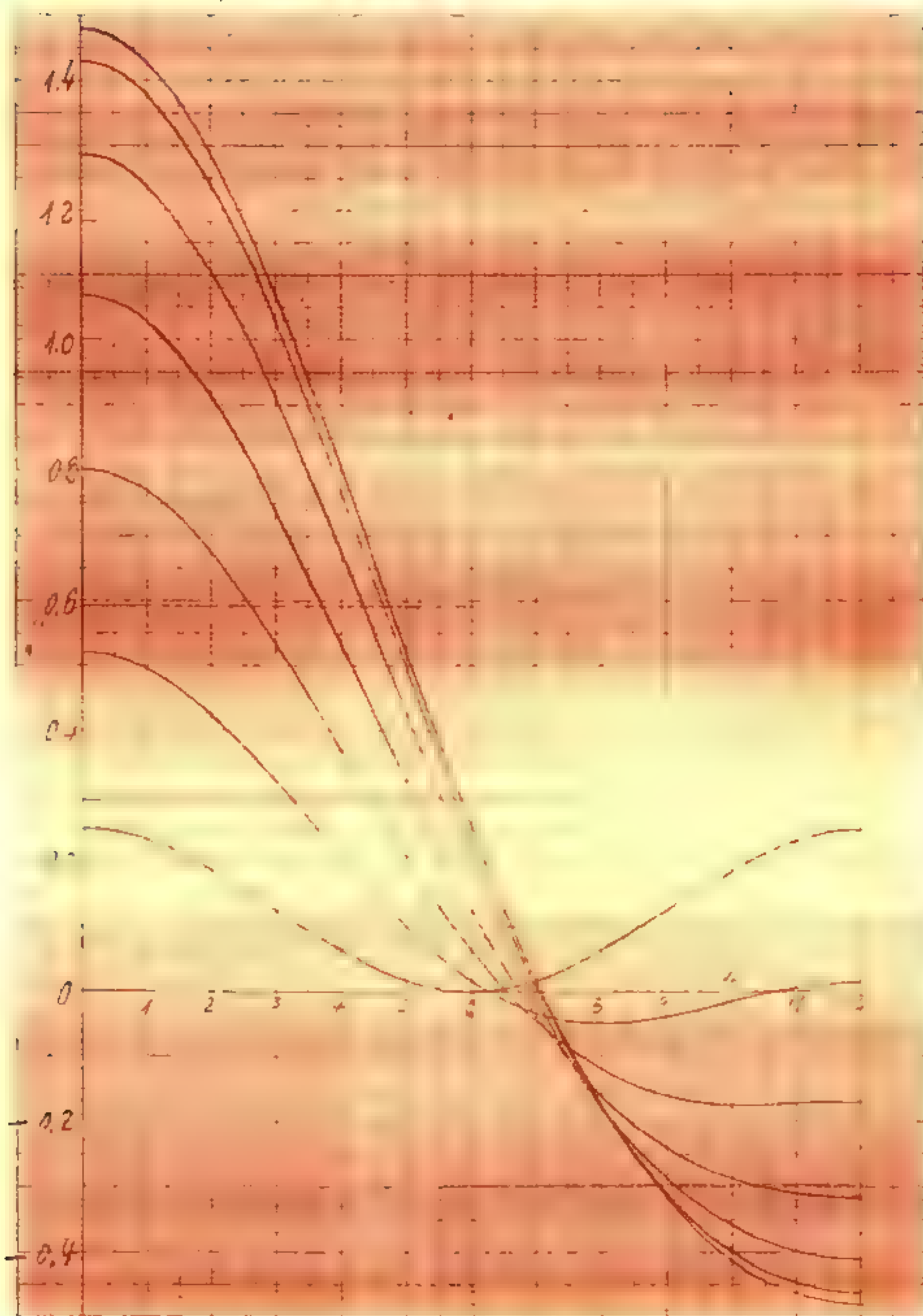
$$\mu=2, \quad n=14; \quad \xi=10$$

$$\rho = -0.2071031; \quad \omega_1: \omega_2 = 1: 0.35354$$

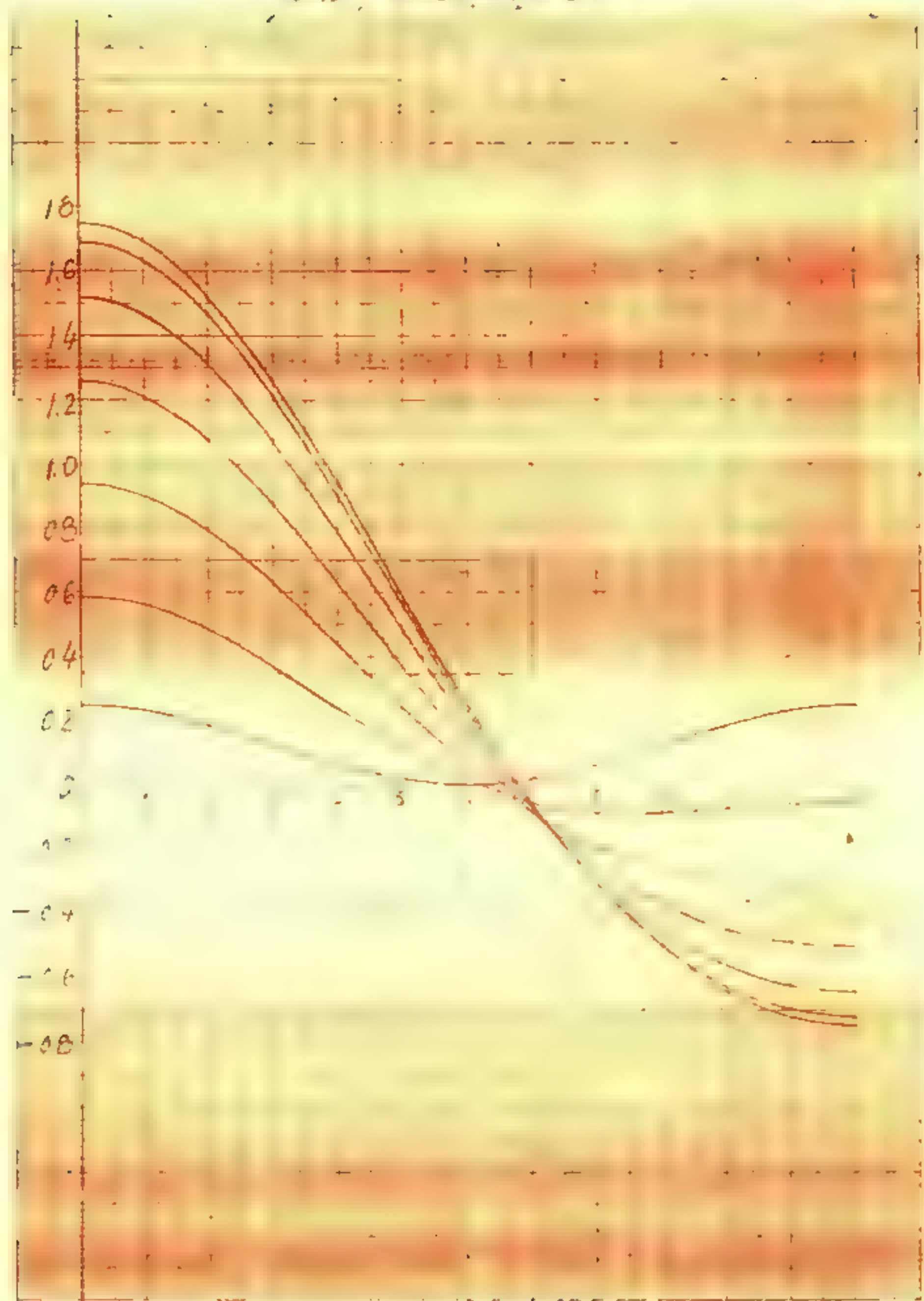
| $\frac{\pi x}{R}$ | η/R | | | | | | |
|-------------------|-----------|------------|------------|------------|------------|------------|------------|
| | 0° | 15° | 30° | 45° | 60° | 75° | 90° |
| 0° | 1.3535 | 1.3075 | 1.1766 | 0.9786 | 0.7393 | 0.4877 | 0.2500 |
| 15° | 1.3075 | 1.2629 | 1.1347 | 0.9413 | 0.7081 | 0.4634 | 0.2333 |
| 30° | 1.1766 | 1.1347 | 1.0152 | 0.8352 | 0.6196 | 0.3955 | 0.1875 |
| 45° | 0.9786 | 0.9413 | 0.8352 | 0.6768 | 0.4893 | 0.2980 | 0.1250 |
| 60° | 0.7393 | 0.7081 | 0.6196 | 0.4893 | 0.3384 | 0.1897 | 0.0625 |
| 75° | 0.4877 | 0.4634 | 0.3955 | 0.2980 | 0.1897 | 0.0907 | 0.0167 |
| 90° | 0.2500 | 0.2333 | 0.1875 | 0.1250 | 0.0625 | 0.0167 | 0 |
| 105° | 0.0458 | 0.0366 | 0.0130 | -0.0144 | -0.0312 | -0.0237 | 0.0167 |
| 120° | -0.1143 | -0.1165 | -0.1196 | -0.1143 | -0.0884 | -0.0312 | 0.0625 |
| 135° | -0.2285 | -0.2247 | -0.2102 | -0.1768 | -0.1143 | -0.0144 | 0.1250 |
| 150° | -0.3017 | -0.2932 | -0.2652 | -0.2102 | -0.1196 | 0.0130 | 0.1875 |
| 165° | -0.3412 | -0.3299 | -0.2932 | -0.2247 | -0.1165 | 0.0366 | 0.2333 |
| 180° | -0.3535 | -0.3412 | -0.3017 | -0.2285 | -0.1143 | 0.0458 | 0.2500 |



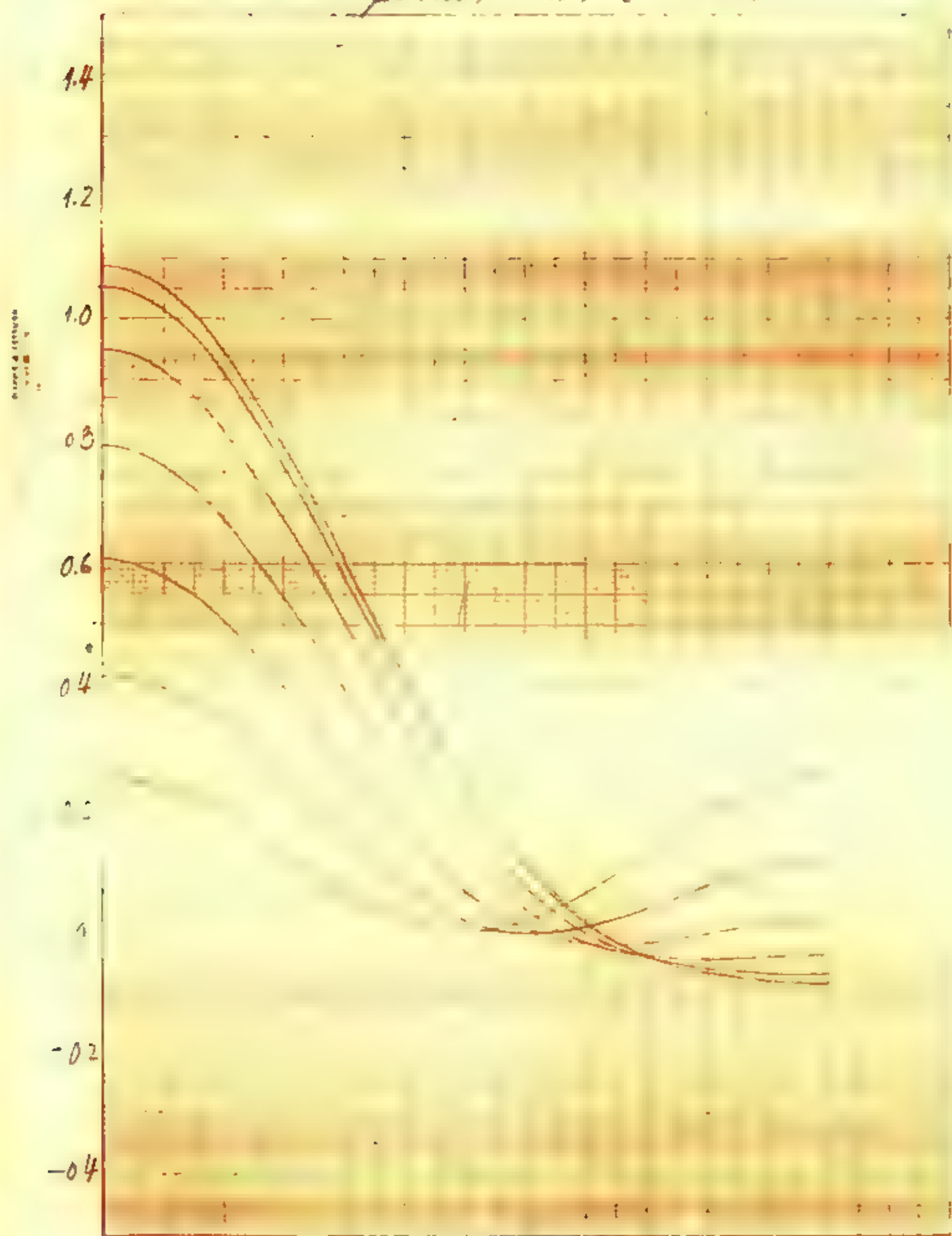
$$\mu = 2.0; \quad n = 17, \quad \xi = 5.5$$



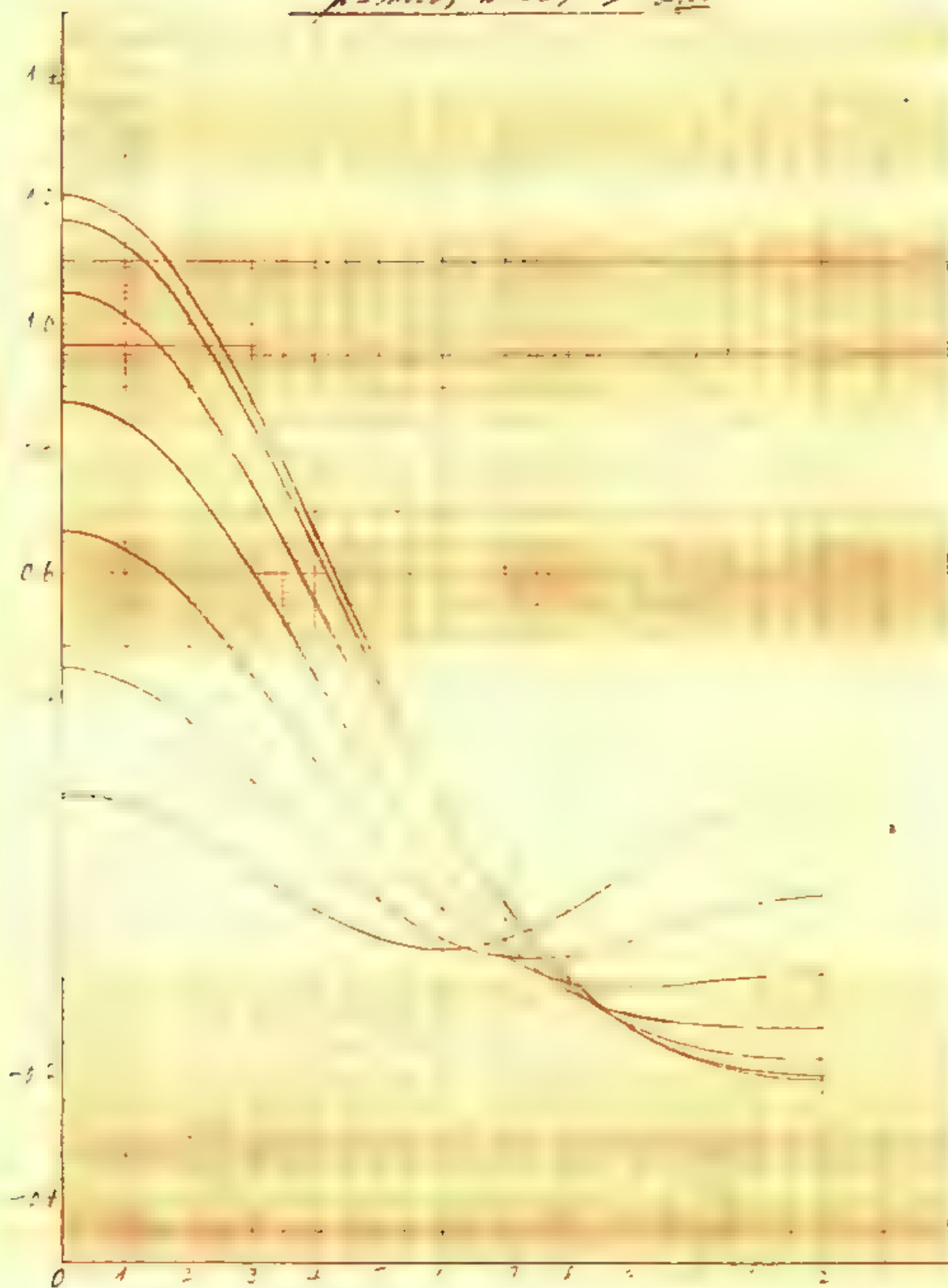
$u = \frac{1}{2}, \quad v = \frac{1}{2}, \quad z = \frac{1}{2}$



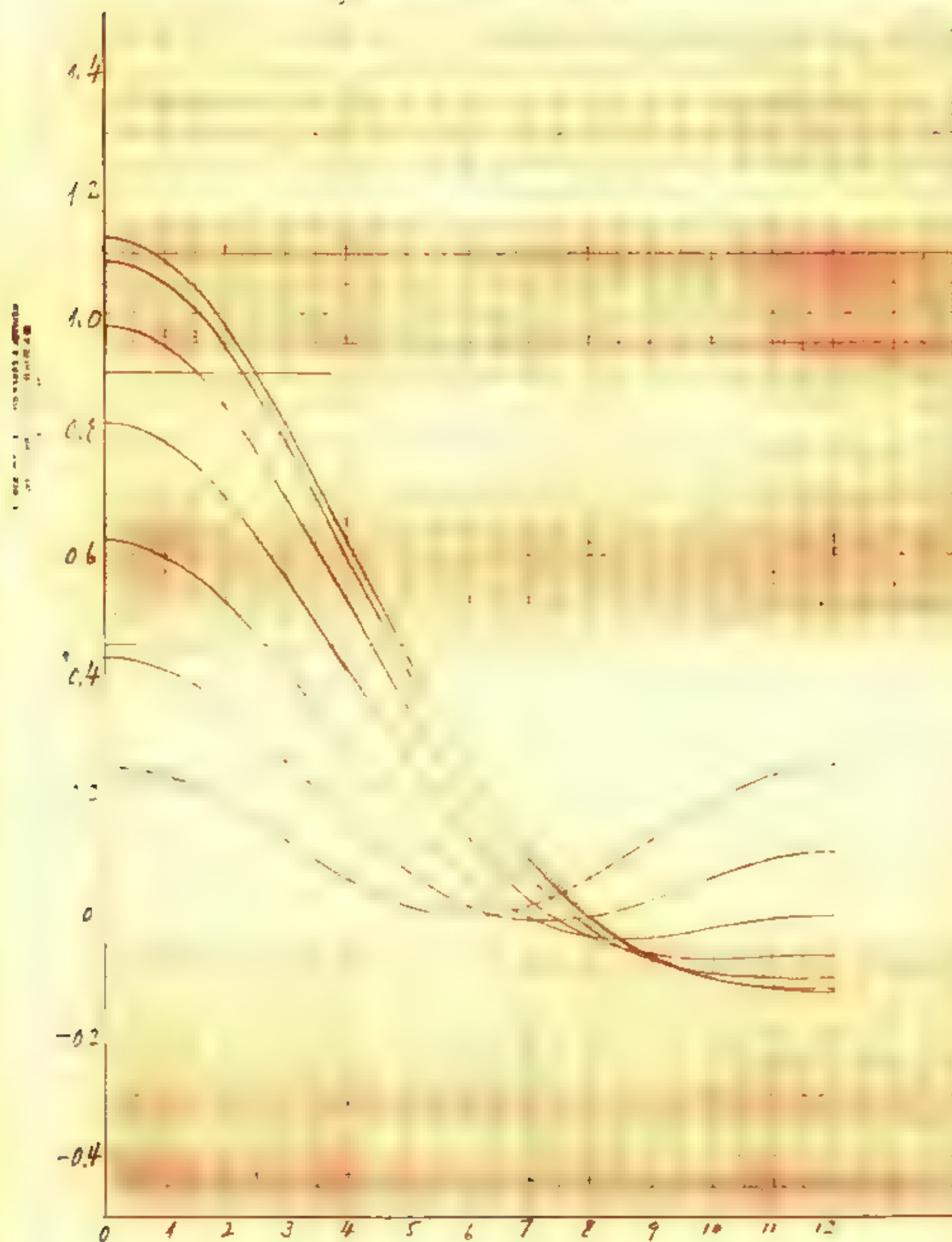
$$\mu = 1000, \quad n = 10; \quad \xi = 16.24518$$

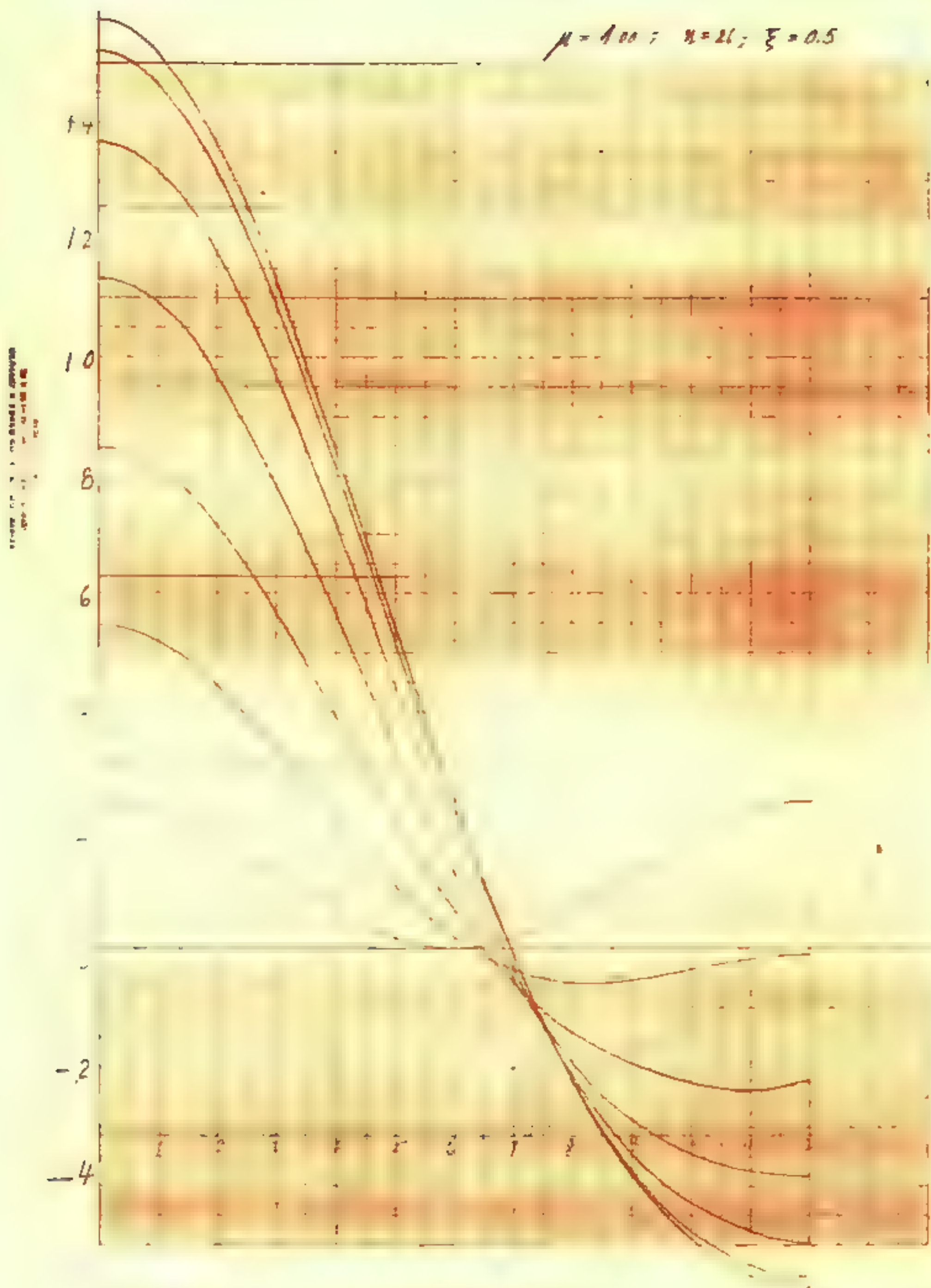


$$\mu = 1.000; n = 26; \bar{E} = 2.05$$

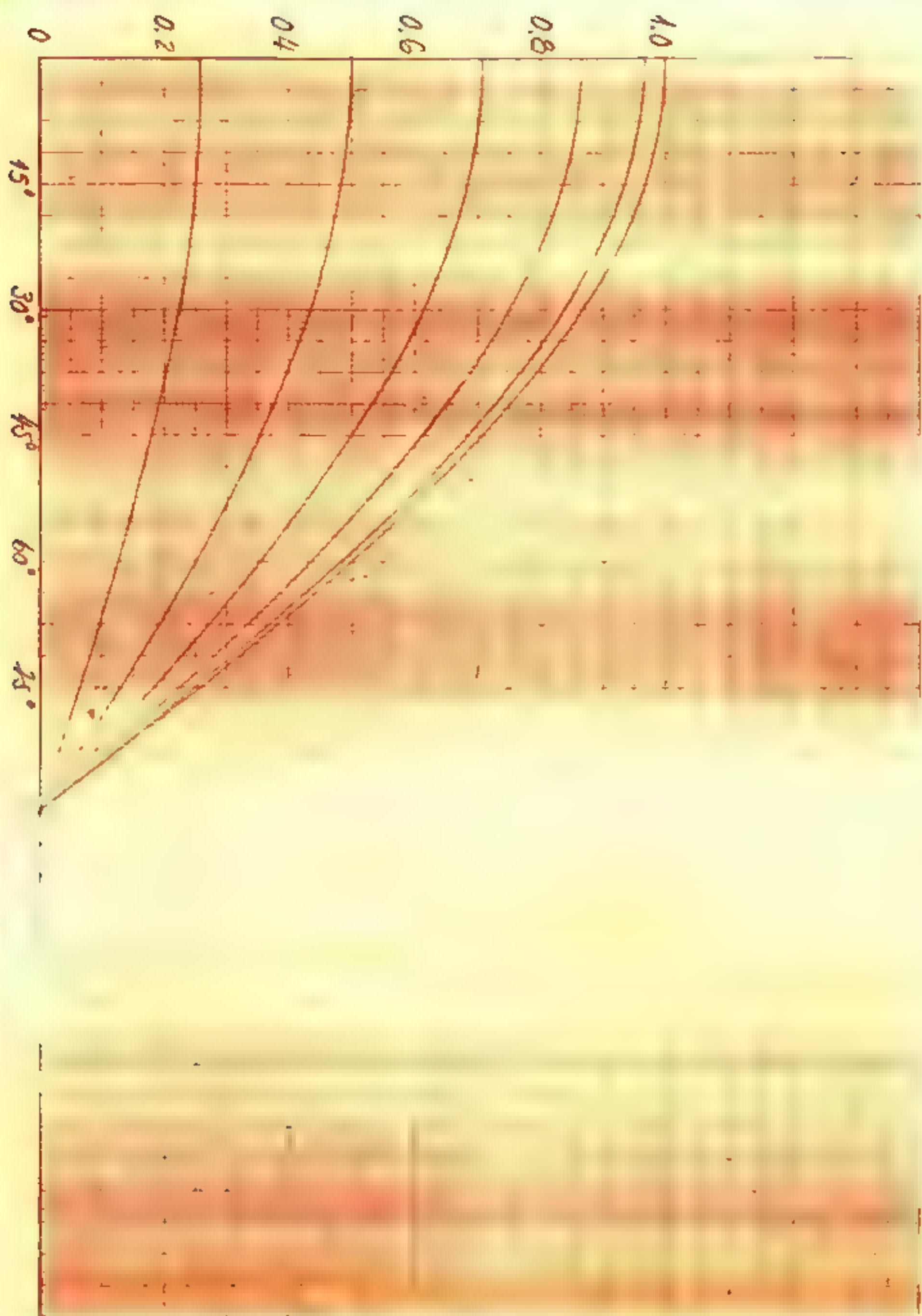


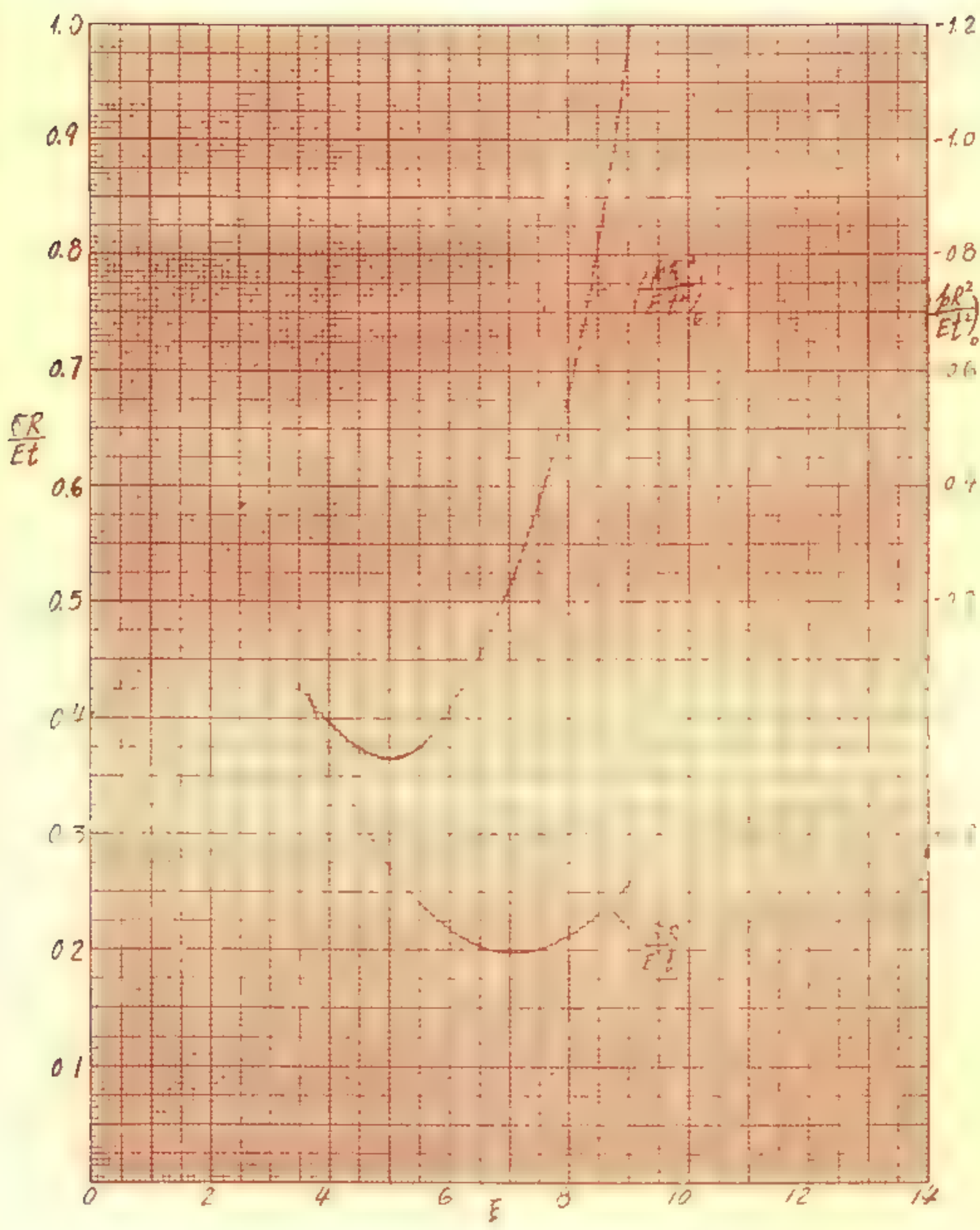
$$n = 1000, \quad n = 20; \quad \bar{z} = 40$$





$\mu = 1.00$; $n = 26$; $\xi = 0$





$$+(\sigma_x + \delta) \frac{\partial^2 w}{\partial x^2} \quad ; \quad + \tau_y \frac{\partial^2 w}{\partial y^2} \quad ; \quad + \tau_{xy} \frac{\partial^2 w}{\partial x \partial y} \quad \underline{a}$$

Here are the forces which will help the buckling

$$+(\sigma_x + \delta) = + E \mu^2 \left[\frac{B}{4} \cos \frac{\pi x}{R} + \frac{C}{(1+\mu^2)^2} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \frac{D}{(1+9\mu^2)^2} \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} \right. \\ \left. + \frac{9G}{(9+\mu^2)^2} \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} + \frac{H}{4(1+\mu^2)^2} \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} \right]$$

$$+ \tau_y = + E \mu^2 \left[\frac{A}{4\mu^2} \cos \frac{2\pi x}{R} + \frac{\mu^2 C}{(1+\mu^2)^2} \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} \right. \\ \left. + \frac{9\mu^2 D}{(1+9\mu^2)^2} \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} + \frac{\mu^2 G}{(9+\mu^2)^2} \cos \frac{\pi x}{R} \cos \frac{3\pi y}{R} + \frac{\mu^2 H}{4(1+\mu^2)^2} \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} \right]$$

$$+ \tau_{xy} = + E \mu^2 \left[\frac{\mu C}{(1+\mu^2)^2} \sin \frac{\pi x}{R} \sin \frac{\pi y}{R} + \frac{3\mu D}{(1+9\mu^2)^2} \cos \frac{3\pi x}{R} \sin \frac{\pi y}{R} + \right. \\ \left. + \frac{3\mu G}{(9+\mu^2)^2} \sin \frac{\pi x}{R} \sin \frac{3\pi y}{R} + \frac{\mu H}{4(1+\mu^2)^2} \sin \frac{2\pi x}{R} \sin \frac{2\pi y}{R} \right]$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{\pi^2}{R} \mu^2 \left\{ \frac{1}{2} f_1 \left[\cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \cos \frac{2\pi x}{R} \right] + f_2 \cos \frac{2\pi x}{R} \right\}$$

$$= -\frac{\pi^2}{R} \mu^2 \left\{ \frac{1}{2} f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \left(\frac{1}{2} f_1 + f_2 \right) \cos \frac{2\pi x}{R} \right\}$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{\pi^2}{R} \left\{ \frac{1}{2} f_1 \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} + \left(\frac{1}{2} f_1 + f_2 \right) \cos \frac{2\pi y}{R} \right\}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\pi^2}{R} \mu \left\{ \frac{1}{2} f_1 \sin \frac{\pi x}{R} \sin \frac{\pi y}{R} \right\}$$

$$+ (G_2 + G_1) \frac{\partial \psi}{\partial y^2} = -\frac{\pi^2}{R^2} E \mu^2 \left[\frac{B}{8} \frac{\partial f}{\partial y} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} + \frac{C f_1 \mu^2}{2(1+\mu^2)^2} \cos^2 \frac{\pi y}{R} \cos \frac{2\pi y}{R} + \frac{D f_1 \mu^2}{2(1+9\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} \cos \frac{2\pi y}{R} \right. \\ \left. + \frac{9 G f_1 \mu^2}{2(9+\mu^2)^2} \cos^2 \frac{\pi y}{R} \cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} + \frac{H f_1 \mu^2}{8(1+\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} + \right.$$

$$\left. + \mu^2 \left(\frac{1}{2} f_1 + f_2 \right) \left\{ \frac{B}{4} \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} + \frac{C}{(1+\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} + \frac{D}{(1+9\mu^2)^2} \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} \right. \right. \\ \left. \left. + \frac{9G}{(9+\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} + \frac{H}{4(1+\mu^2)^2} \cos^2 \frac{2\pi y}{R} \cos \frac{2\pi y}{R} \right\} \right]$$

$$+ G_2 \frac{\partial \psi}{\partial y^2} = -\frac{\pi^2}{R^2} E \mu^2 \left[\frac{A f_1 \mu^2}{8 \mu^6} \cos \frac{\pi y}{R} \cos \frac{\pi y}{R} \cos \frac{\pi y}{R} + \frac{C f_1 \mu^2}{2(1+\mu^2)^2} \cos^2 \frac{\pi y}{R} \cos \frac{2\pi y}{R} + \frac{9 D f_1 \mu^2}{2(1+9\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} \right.$$

$$\left. + \frac{G f_1 \mu^2}{2(9+\mu^2)^2} \cos^2 \frac{\pi y}{R} \cos \frac{\pi y}{R} \cos \frac{3\pi y}{R} + \frac{H f_1 \mu^2}{8(1+\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} + \right.$$

$$\left. + \mu^2 \left(\frac{1}{2} f_1 + f_2 \right) \left\{ \frac{A}{4 \mu^6} \cos \frac{2\pi y}{R} \cos \frac{2\pi y}{R} + \frac{C}{(1+\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} + \frac{9D}{(1+9\mu^2)^2} \cos \frac{3\pi y}{R} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} \right. \right. \\ \left. \left. + \frac{G}{(9+\mu^2)^2} \cos \frac{\pi y}{R} \cos \frac{2\pi y}{R} \cos \frac{3\pi y}{R} + \frac{H}{4(1+\mu^2)^2} \cos^2 \frac{2\pi y}{R} \cos \frac{2\pi y}{R} \right\} \right]$$

$$\begin{aligned}
+ 2 E_{xy} \frac{\partial u}{\partial y} &= \frac{\mu^2}{R^2} E \mu^2 \left[\frac{2 \mu^2 C_1}{2(1+\mu^2)} \sin^2 \frac{m\pi}{R} - \frac{6 \mu^2 D_1}{2(1+9\mu^2)} \sin \frac{m\pi}{R} \cos \frac{3m\pi}{R} \sin^2 \frac{m\pi}{R} \right. \\
&\quad \left. - \frac{6 \mu^2 G_1}{2(9+\mu^2)^2} \sin^2 \frac{m\pi}{R} \sin \frac{m\pi}{R} \cos \frac{3m\pi}{R} - \frac{2 \mu^2 H_1}{F(1+\mu^2)^2} \sin \frac{m\pi}{R} \sin \frac{2m\pi}{R} \sin \frac{m\pi}{R} \cos \frac{2m\pi}{R} \right] \\
\phi &= -\frac{\mu^2}{R^2} E \mu^4 \left[\frac{B_1}{8} \cos \frac{m\pi}{R} \cos \frac{m\pi}{R} \cos \frac{2m\pi}{R} + \frac{A_1}{8 \mu^4} \cos \frac{m\pi}{R} \cos \frac{2m\pi}{R} \cos \frac{m\pi}{R} + \frac{C_1}{(1+\mu^2)^2} \left(\cos^3 \frac{m\pi}{R} \cos \frac{2m\pi}{R} \right. \right. \\
&\quad \left. \left. + \frac{D_1}{(1+9\mu^2)^2} \left(5 \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} \cos \frac{m\pi}{R} - 3 \sin \frac{m\pi}{R} \sin \frac{3m\pi}{R} \sin \frac{m\pi}{R} \right) \right. \right. \\
&\quad \left. \left. + \frac{G_1}{(9+\mu^2)^2} \left(15 \cos^3 \frac{m\pi}{R} \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} - 3 \sin^2 \frac{m\pi}{R} \sin \frac{m\pi}{R} \cos \frac{3m\pi}{R} \right) \right. \right. \\
&\quad \left. \left. + \frac{H_1}{4(1+\mu^2)^2} \left(\cos \frac{m\pi}{R} \cos \frac{2m\pi}{R} \cos \frac{m\pi}{R} \cos \frac{m\pi}{R} - \sin \frac{m\pi}{R} \sin \frac{2m\pi}{R} \sin \frac{m\pi}{R} \cos \frac{2m\pi}{R} \right) \right. \right. \\
&\quad \left. \left. + \left(\frac{1}{2} A_1 + \frac{1}{2} B_1 \right) \left(\frac{1}{4} \left(\frac{1}{\mu^2} + 8 \right) \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} + \frac{C}{(1+\mu^2)^2} \left(\cos \frac{m\pi}{R} \cos \frac{m\pi}{R} + \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} \right) \right) \right. \right. \\
&\quad \left. \left. + \frac{D}{(1+9\mu^2)^2} \left[\cos \frac{2m\pi}{R} \cos \frac{3m\pi}{R} \cos \frac{m\pi}{R} + 9 \cos \frac{2m\pi}{R} \cos \frac{m\pi}{R} \cos \frac{3m\pi}{R} \right] + \frac{G}{(9+\mu^2)^2} \left[9 \cos \frac{m\pi}{R} \cos \frac{2m\pi}{R} \cos \frac{3m\pi}{R} + \cos \frac{m\pi}{R} \cos \frac{2m\pi}{R} \cos \frac{3m\pi}{R} \right] \right. \right. \\
&\quad \left. \left. + \frac{H}{4(1+\mu^2)^2} \left[\cos^2 \frac{2m\pi}{R} \cos \frac{2m\pi}{R} + \cos \frac{2m\pi}{R} \cos \frac{2m\pi}{R} \right] \right] \right]
\end{aligned}$$

$$\begin{aligned}
\gamma_0 = & \frac{1}{R} \frac{1}{(1+\mu^2)^2} E \mu^4 \left[\frac{B}{16} \cos \frac{m\pi}{R} \left(\cos \frac{n\pi}{R} + \cos \frac{3n\pi}{R} \right) + \frac{A}{16\mu^4} \left(\cos \frac{2n\pi}{R} + \cos \frac{3m\pi}{R} \right) \cos \frac{n\pi}{R} + \frac{C}{2(1+\mu^2)^2} \left(\cos \frac{2m\pi}{R} + \cos \frac{2n\pi}{R} \right) \right. \\
& + \frac{D}{(1+9\mu^2)^2} \left\{ \frac{1}{2} \left(\cos \frac{1m\pi}{R} + \cos \frac{4m\pi}{R} \cos \frac{2n\pi}{R} \right) + 2 \left(\cos \frac{4n\pi}{R} + \cos \frac{2m\pi}{R} \cos \frac{2n\pi}{R} \right) \right\} \\
& + \frac{G}{(9+\mu^2)^2} \left\{ \frac{1}{2} \left(\cos \frac{2n\pi}{R} + \cos \frac{2m\pi}{R} \cos \frac{4n\pi}{R} \right) + 2 \left(\cos \frac{4n\pi}{R} + \cos \frac{2m\pi}{R} \cos \frac{2n\pi}{R} \right) \right\} \\
& + \frac{H}{8(1+\mu^2)^2} \left\{ \cos \frac{3m\pi}{R} \cos \frac{n\pi}{R} + \cos \frac{n\pi}{R} \cos \frac{3n\pi}{R} \right\} + \left(\frac{1}{2} + \frac{B}{2} \right) \left\{ \frac{1}{4} \left(\frac{A}{R} + \frac{B}{R} \right) \cos \frac{2m\pi}{R} \cos \frac{2n\pi}{R} + \right. \\
& + \frac{C}{2(1+\mu^2)^2} \left[\left(\cos \frac{2n\pi}{R} + \cos \frac{3m\pi}{R} \right) \cos \frac{n\pi}{R} + \cos \frac{n\pi}{R} \left(\cos \frac{n\pi}{R} + \cos \frac{3n\pi}{R} \right) \right] \\
& + \frac{D}{2(1+9\mu^2)^2} \left[\left(\cos \frac{n\pi}{R} + \cos \frac{5m\pi}{R} \right) \cos \frac{n\pi}{R} + 9 \cos \frac{3m\pi}{R} \left(\cos \frac{n\pi}{R} + \cos \frac{3n\pi}{R} \right) \right] \\
& + \frac{G}{2(9+\mu^2)^2} \left[9 \left(\cos \frac{m\pi}{R} + \cos \frac{3n\pi}{R} \right) \cos \frac{3n\pi}{R} + \cos \frac{m\pi}{R} \left(\cos \frac{n\pi}{R} + \cos \frac{5n\pi}{R} \right) \right] \\
& + \frac{H}{8(1+\mu^2)^2} \left[\left(1 + \cos \frac{4n\pi}{R} \right) \cos \frac{n\pi}{R} + \cos \frac{2m\pi}{R} \left(1 + \cos \frac{4n\pi}{R} \right) \right] + \frac{1}{R} E \mu^4 \left[\frac{A}{4\mu^4} \cos \frac{2m\pi}{R} + \right. \\
& \left. + \frac{C}{(1+\mu^2)^2} \cos \frac{m\pi}{R} \cos \frac{n\pi}{R} + \frac{9D}{(1+9\mu^2)^2} \cos \frac{3m\pi}{R} \cos \frac{n\pi}{R} + \frac{G}{(9+\mu^2)^2} \cos \frac{2n\pi}{R} \cos \frac{3m\pi}{R} + \frac{H}{4(1+\mu^2)^2} \cos \frac{2m\pi}{R} \cos \frac{2n\pi}{R} \right]
\end{aligned}$$

$$\begin{aligned}
\frac{kR}{tE} \frac{1}{\mu^4} = & \cos \frac{2\pi x}{R} \left\{ -\eta \xi \left(\frac{C^v}{2(1+\mu^2)^2} + \frac{1}{2} \frac{E}{(1+\mu^2)^2} + \frac{H}{8(1+\mu^2)^2} \left(\frac{1}{2} + S \right) \right) + \frac{A}{4\mu^4} \right\} \frac{e}{\mu^4} \\
& + \cos \frac{2\pi y}{R} \left\{ -\eta \xi \left(\frac{C^v}{2(1+\mu^2)^2} + \frac{1}{2} \frac{G}{(1+\mu^2)^2} + \frac{H}{8(1+\mu^2)^2} \left(\frac{1}{2} + S \right) \right) \right\} \\
& + \cos \frac{4\pi x}{R} \left\{ -\eta \xi \left(\frac{2D}{(1+\mu^2)^2} \right) \right\} + \cos \frac{4\pi y}{R} \left\{ -\eta \xi \left(\frac{2G}{(1+\mu^2)^2} \right) \right\} \\
& + \cos \frac{\pi x}{R} \cos \frac{\pi y}{R} \left\{ -\eta \xi \left(\frac{B}{16} + \frac{A}{16\mu^4} + \left(\frac{1}{2} + S \right) \left(\frac{C^v}{(1+\mu^2)^2} + \frac{D}{2(1+\mu^2)^2} + \frac{G}{2(1+\mu^2)^2} \right) \right. \right. \\
& \quad \left. \left. + \frac{C}{(1+\mu^2)^2} \right) \right\} \\
& + \cos \frac{\pi y}{R} \cos \frac{3\pi x}{R} \left\{ -\eta \xi \left(\frac{B}{16} + \frac{H}{8(1+\mu^2)^2} + \left(\frac{1}{2} + S \right) \left(\frac{C^v}{2(1+\mu^2)^2} + \frac{9G}{2(1+\mu^2)^2} \right) \right) + \frac{G}{(1+\mu^2)^2} \right\} \\
& + \cos \frac{3\pi x}{R} \cos \frac{\pi y}{R} \left\{ -\eta \xi \left(\frac{A}{16\mu^4} + \frac{H}{8(1+\mu^2)^2} + \left(\frac{1}{2} + S \right) \left(\frac{C^v}{2(1+\mu^2)^2} + \frac{9D}{2(1+\mu^2)^2} \right) \right) + \frac{9D}{(1+\mu^2)^2} \right\} \\
& + \cos \frac{2\pi x}{R} \cos \frac{2\pi y}{R} \left\{ -\eta \xi \left(\frac{2D}{(1+\mu^2)^2} + \frac{2G}{(1+\mu^2)^2} + \left(\frac{1}{2} + S \right) \frac{1}{4} \left(\frac{A}{\mu^4} + B \right) \right) + \frac{H}{4(1+\mu^2)^2} \right\} \\
& + \cos \frac{\pi x}{R} \cos \frac{5\pi y}{R} \left\{ -\left(\frac{1}{2} + S \right) \eta \xi \frac{G}{2(1+\mu^2)^2} \right\} + \cos \frac{5\pi x}{R} \cos \frac{\pi y}{R} \left\{ -\left(\frac{1}{2} + S \right) \eta \xi \frac{D}{2(1+\mu^2)^2} \right\} \\
& + \cos \frac{2\pi x}{R} \cos \frac{4\pi y}{R} \left\{ \frac{G}{2(1+\mu^2)^2} + \left(\frac{1}{2} + S \right) \frac{H}{8(1+\mu^2)^2} \right\} \eta \xi + \cos \frac{4\pi x}{R} \cos \frac{2\pi y}{R} \left\{ \frac{D}{2(1+\mu^2)^2} + \left(\frac{1}{2} + S \right) \frac{H}{8(1+\mu^2)^2} \right\} \eta \xi \\
& + \cos \frac{3\pi x}{R} \cos \frac{3\pi y}{R} \left\{ \frac{9D}{2(1+\mu^2)^2} + \frac{9G}{2(1+\mu^2)^2} \right\} \left(\frac{1}{2} + S \right) \eta \xi
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\partial R}{\partial E} \frac{1}{\mu^4}\right)_0 &= -\eta \xi \left[\frac{\check{B}}{8} + \frac{\check{A}}{8\mu^4} + \frac{C}{(1+\mu^2)^2} + \frac{5D}{(1+9\mu^2)^2} + \frac{5G}{(9+\mu^2)^2} + \frac{H}{4(1+\mu^2)^2} \right] \\
&+ \left(\frac{1}{2} + \eta\right) \left\{ \frac{1}{4} \left(\frac{\check{A}}{\mu^4} + \check{B} \right) + \frac{2C}{(1+\mu^2)^2} + \frac{10D}{(1+9\mu^2)^2} + \frac{10G}{(9+\mu^2)^2} + \frac{H}{2(1+\mu^2)^2} \right\} \\
&+ \frac{\check{A}}{4\mu^4} + \frac{C}{(1+\mu^2)^2} + \frac{9D}{(1+9\mu^2)^2} + \frac{G}{(9+\mu^2)^2} + \frac{H}{4(1+\mu^2)^2} \\
&= \frac{A}{4\mu^4} \left[-(1+\eta) \eta \xi + 1 \right] + \frac{B}{4} \left[-(1+\eta) \eta \xi \right] + \frac{C}{(1+\mu^2)^2} \left[-2(1+\eta) \eta \xi + 1 \right] \\
&+ \frac{D}{(1+9\mu^2)^2} \left[10(1+\eta) \eta \xi + 9 \right] + \frac{G}{(9+\mu^2)^2} \left[10(1+\eta) \eta \xi + 1 \right] + \frac{H}{4(1+\mu^2)^2} \left[-2(1+\eta) \eta \xi + 1 \right]
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\partial R^2}{\partial E^2} \frac{1}{\mu^4}\right)_0 &= \frac{\xi \left\{ \frac{1}{8} \eta \xi - \left(\frac{1}{2} + \eta\right) \right\}}{4\mu^4} \left[-(1+\eta) \eta \xi + 1 \right] + \frac{\xi \left\{ \frac{1}{8} \eta \xi \right\}}{4} \left[-(1+\eta) \eta \xi \right] \\
&+ \frac{\xi \left\{ \frac{1}{2} \left(\frac{1}{2} + \eta\right) \eta \xi - \frac{1}{2} \right\}}{(1+\mu^2)^2} \left[-2(1+\eta) \eta \xi + 1 \right] + \frac{\frac{1}{4} \xi \left(\frac{1}{2} + \eta\right) \eta \xi}{(1+9\mu^2)^2} \left[-10(1+\eta) \eta \xi + 9 \right] \\
&+ \frac{\frac{1}{4} \xi \left(\frac{1}{2} + \eta\right) \eta \xi}{(9+\mu^2)^2} \left[10(1+\eta) \eta \xi + 1 \right] + \frac{\xi \left(\frac{1}{2} + \eta\right)^2 \eta}{4(1+\mu^2)^2} \left[-2(1+\eta) \eta \xi + 1 \right] \\
&= \mathcal{F}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\rho R^2}{E t^2} \frac{1}{\mu^4} \right)_0 &= \xi \left[-(\eta \xi)^2 \left\{ \frac{1+\rho}{32\mu^4} + \frac{1+\rho}{32} + \frac{(1+\rho)(\frac{1}{2}+\rho)}{(1+\mu^2)^2} + \frac{5(1+\rho)(\frac{1}{2}+\rho)}{2(1+\mu^2)^2} \right. \right. \\
&\quad \left. \left. + \frac{5(1+\rho)(\frac{1}{2}+\rho)}{2(1+\mu^2)^2} + \frac{(1+\rho)(\frac{1}{2}+\rho)^2}{2(1+\mu^2)^2} \right\} \right. \\
&\quad + (\eta \xi) \left\{ \frac{1}{32\mu^4} + \frac{(1+\rho)(\frac{1}{2}+\rho)}{4\mu^4} + \frac{(\frac{1}{2}+\rho)}{2(1+\mu^2)^2} + \frac{(1+\rho)}{(1+\mu^2)^2} + \frac{9(\frac{1}{2}+\rho)}{4(1+\mu^2)^2} \right. \\
&\quad \left. \left. + \frac{(\frac{1}{2}+\rho)}{4(1+\mu^2)^2} + \frac{(\frac{1}{2}+\rho)^2}{4(1+\mu^2)^2} \right\} - \left\{ \frac{(\frac{1}{2}+\rho)}{4\mu^4} + \frac{1}{2(1+\mu^2)^2} \right\} \right] \\
&= \xi \left[-(\eta \xi)^2 (1+\rho) \left\{ \frac{1}{32\mu^4} + \frac{1}{32} + \frac{(\frac{1}{2}+\rho)}{(1+\mu^2)^2} + \frac{5(\frac{1}{2}+\rho)}{2(1+\mu^2)^2} + \frac{5(\frac{1}{2}+\rho)}{2(1+\mu^2)^2} + \frac{(\frac{1}{2}+\rho)^2}{2(1+\mu^2)^2} \right\} \right. \\
&\quad + (\eta \xi) \left\{ \frac{1}{32\mu^4} + \frac{(1+\rho)}{(1+\mu^2)^2} + (\frac{1}{2}+\rho) \left[\frac{1}{2(1+\mu^2)^2} + \frac{(1+\rho)}{4\mu^4} + \frac{9}{4(1+\mu^2)^2} + \frac{1}{4(1+\mu^2)^2} \right. \right. \\
&\quad \left. \left. + \frac{(\frac{1}{2}+\rho)}{4(1+\mu^2)^2} \right] \right\} - \left\{ \frac{(\frac{1}{2}+\rho)}{4\mu^4} + \frac{1}{2(1+\mu^2)^2} \right\} \right]
\end{aligned}$$

$$\eta = 0.225 ; \quad \xi = 6 ; \quad \rho = -0.0706506$$

2

$$\eta\xi = 1.350$$

$$1+\rho = 0.9291494$$

$$(\eta\xi)^2 = 1.8225$$

$$\frac{1}{2} + \rho = 0.4291494$$

$$\left(\frac{\rho R^2}{Et^2}\right)_0 = 6 \left[-1.8225 \times 0.9291494 \times 0.2142660 + 1.350 \left(\overset{0.4391064}{0.3279098 + 0.1111966} \right) - 0.2322874 \right]$$

$$= -0.0139584$$

$$\eta = 0.225 ; \quad \xi = 7.5 ; \quad \rho = -0.0452641$$

$$\eta\xi = 1.6875$$

$$1+\rho = 0.9147359$$

$$(\eta\xi)^2 = 2.84765625$$

$$\frac{1}{2} + \rho = 0.4147359$$

$$\left(\frac{\rho R^2}{Et^2}\right)_0 = 7.5 \left[-2.84765625 \times 0.9147359 \times 0.2084215 + 1.6875 \left(\overset{0.4277382}{\underset{0.1655938}{0.3221444 + 0.1055938}} \right) - 0.2286840 \right]$$

$$= -0.3733740$$

$$\eta = 0.225 ; \quad \xi = 9 ; \quad \rho = -0.1008270 ;$$

$$\eta\xi = 2.025$$

$$1+\rho = 0.8991730$$

$$(\eta\xi)^2 = 4.100625$$

$$\frac{1}{2} + \rho = 0.3991730$$

$$\left(\frac{\rho R^2}{Et^2}\right)_0 = 9 \left[-4.100625 \times 0.8991730 \times 0.2021693 + 2.025 \left(\overset{0.4156093}{\underset{0.0995901}{0.3159192 + 0.0995901}} \right) - 0.2247933 \right]$$

$$= -1.1575557$$

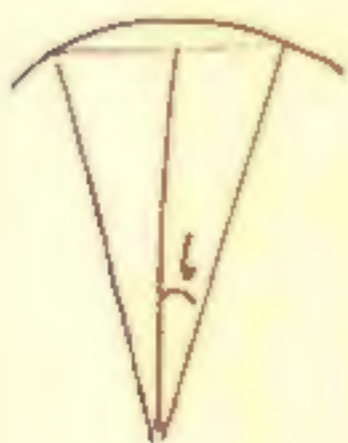
When $\mu = 2$.

$$\left(\frac{\rho R^2}{Et^2}\right)_0 = 3 \left[-(\eta\xi)^2(1+\rho) \left\{ \frac{1}{16} + \frac{3(\frac{1}{2}+\rho)}{10} + \frac{(\frac{1}{2}+\rho)^2}{8} \right\} + \right. \\ \left. + (\eta\xi) \left\{ \frac{1}{32} + \frac{(1+\rho)}{4} + \frac{3(\frac{1}{2}+\rho)}{20} + \frac{(1+\rho)(\frac{1}{2}+\rho)}{4} + \frac{(\frac{1}{2}+\rho)^2}{16} \right\} \right. \\ \left. - \left\{ \frac{(\frac{1}{2}+\rho)}{4} + \frac{1}{8} \right\} \right]$$

$\eta = 0.225$; $3 = 4$

$(1+\rho) = 0.9450611$, $(\frac{1}{2}+\rho) = 0.4450611$

$$\left(\frac{\rho R^2}{Et^2}\right)_0 = 4 \left[-0.810 \times 0.9450611 \times 0.2207762 + 0.900 \times (0.03125 + 0.2362653 \right. \\ \left. + 0.0667592 + 0.1051525 + 0.0123800) - 0.2362653 \right] \\ = 4 \left[-0.1670056 + 0.4066243 - 0.2362653 \right] = \underline{0.0054216}$$



$1 - \cos 6^\circ = 0.0055$

$\theta = \frac{2\pi R}{15}$

$\frac{t}{R} = \frac{2\pi}{15} \frac{R}{t}$

$\frac{\rho}{E} =$